# 1 Supplement

## 1.1 Momentum Correction, Warm-up Training and Momentum Masking:

Lin et al. [17] introduce multiple minor modifications to the vanilla Gradient Dropping method. With these modifications they achieve up to around 1% higher accuracy compared to Gradient Dropping on a variety of benchmarks. Those modifications include:

*Momentum correction*: Instead of adding the raw gradient to the residuum, the momentum-corrected gradient is added. This is used implicitly in our approach, as our weight updates are already momentum-corrected.
*Warm-up Training*: The sparsity rate is increased exponentially from 25% to 0.1% in the first epochs. We find that warm-up training can indeed speed-up convergence in the beginning of training, but ultimately has no effect on the final accuracy of the model. We therefore omit warm up training in our experiments, as it adds an additional hyperparameter to the method, without any real benefit.
*Momentum Masking*: To avoid stale momentum from carrying the optimization into a wrong direction after a weight update is performed, Lin et al. suggest to set the momentum to zero for updated weights. We adopt momentum correction in our method.

## 1.2 Golomb Position Decoding

Algorithm 1 describes the decoding of a binary sequence produced by Golomb Position Encoding (see main paper). Since the shapes of all weight-tensors are known to both the server and all clients, we can omit the shape information in both encoding and decoding.

**Algorithm 1:** Golomb Position Decoding

**input:** binary message msg, bitsize $\mathbf{b}^*$, mean value $\mu$
**output:** sparse tensor $\Delta W^*$
**init:** $\Delta W^* \leftarrow 0 \in \mathbb{R}^n$
• $i \leftarrow 0; q \leftarrow 0; j \leftarrow 0$
**while** $i < \textit{size}(\text{msg})$ **do**
  **if** $\text{msg}[i] = 0$ **then**
    • $j \leftarrow j + q2^{\mathbf{b}^*} + \text{int}_{\mathbf{b}^*}(\text{msg}[i+1], .., \text{msg}[i+\mathbf{b}^*]) + 1$
    • $\Delta W_j^* \leftarrow \mu$
    • $q \leftarrow 0; i \leftarrow i + \mathbf{b}^* + 1$
  **else**
    • $q \leftarrow q + 1; i \leftarrow i + 1$
  **end**
**end**
**return** $\Delta W^*$

## 1.3 Model Specification

Below, we describe the neural network models used in our experiments. Table 1 list the training hyperparameters that were used.

| | Iterations | Optimizer | Batchsize | LR | LR Decay |
|---|---|---|---|---|---|
| LeNet5-Caffe | 2000 | Adam [4] | 128×4 | 0.001 | - |
| ResNet32 | 60000 | Momentum @0.9 | 128×4 | 0.01 | 0.1 @ 30000, 50000 |
| ResNet50 | 700000 | Momentum @0.9 | 32×4 | 0.1 | 0.1 @ 300000, 600000 |
| WordLSTM | 60000 | Gradient Descent | 5×4 | 1.0 | 0.8 @ $24000 + 1200 \times n$ |
| CharLSTM | 16000 | Gradient Descent | 5×4 | 1.0 | 0.8 @ 5000, 8000, 10000, 12000, 14000 |

Table 1: Hyperparameters used for our experiments in sections 3 and 4.

**LeNet5-Caffe**: The model specification can be downloaded from the Caffe MNIST tutorial page: `https://github.com/BVLC/caffe/blob/master/examples/mnist/lenet_train_test.`

`prototxt`. (Features convolutional layers, fully connected layers, pooling.)

**ResNet32, ResNet50**: We use the implementation from the official Tensorflow repository: `https://github.com/tensorflow/models/tree/master/research/resnet`. (Features skip-connections, batch-normalization.)

**WordLSTM**: We use the implementation from the official Tensorflow repository (configuration "medium"): `https://github.com/tensorflow/models/tree/master/tutorials/rnn/ptb`. (Features trainable word-embeddings, multilayer LSTM-cells, dropout.)

**CharLSTM**: We adapt the implementation of WordLSTM to use a smaller vocabulary of 98 symbols by decreasing the size of the embedding. (Features trainable word-embeddings, multilayer LSTM-cells, dropout.)

### 1.4 Proof of Theorem 2.1.

*Proof.* It holds that

$$\mathrm{err}(\mathcal{R}_{T-1} + \Delta W_T) = \| \sum_{t=1}^{T} \Delta W_t - \sum_{t=1}^{T-1} \Delta W_t^* - \mathcal{R}_{T-1} - \Delta W_T \| = 0. \tag{1}$$

Since $\mathcal{S}$ is a metric subspace, the projection

$$\Delta W_T^* = \mathrm{Proj}_{\mathcal{S}}(\mathcal{R}_{T-1} + \Delta W_T) \tag{2}$$

uniquely solves the minimization problem in $\mathcal{S}$. □

### 1.5 Additional Results

Figure 1 shows convergence speed in terms of iterations (left) and communicated bits (right) respectively for ResNet32 trained on CIFAR. Sparse Binary Compression can train the model to approximately baseline accuracy, using significantly less bits. SBC (3) trains the model to almost baseline accuracy (0.4% degradation), while using $\times 32300$ less bits (cf. table 2).

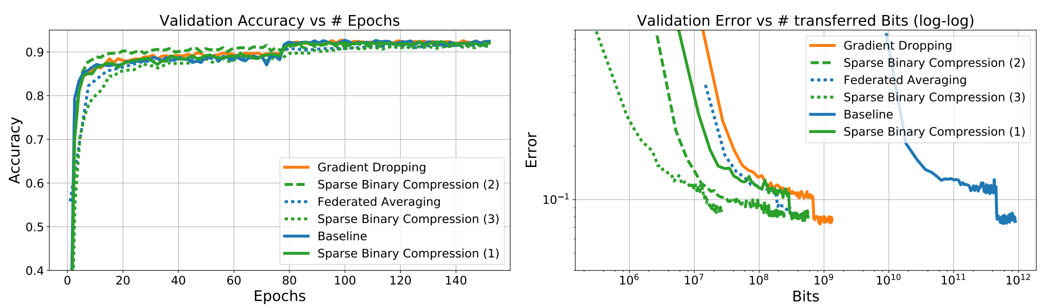


Figure 1: Left: Top-1-Accuracy vs number of epochs. Right: Top-1-error vs transferred number of bits. Log-log plot. ResNet32 on CIFAR.

Figure 2 shows convergence speed in terms of iterations (left) and communicated bits (right) respectively for CharLSTM trained on Shakespeare. Sparse Binary Compression can train the model to approximately baseline accuracy, while using significantly less bits. SBC (1) even achieves the highest accuracy using $\times 2572$ less bits than the baseline (cf. table 2). SBC (3) however, shows non-monotonic convergence behavior on this benchmark. It might be that SBC (3) falls below the total communication budget necessary for this learning task (cf. section 3).

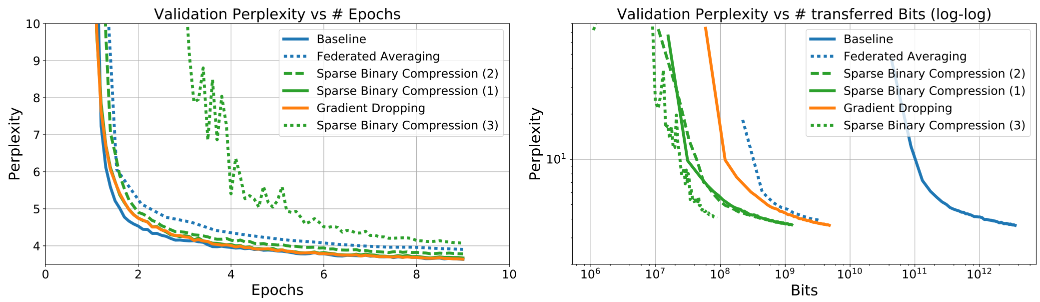


Figure 2: Left: Perplexity vs number of epochs. Right: Perplexity vs transferred number of bits. Log-log plot. CharLSTM on Shakespeare.

Figure 3 shows validation error for WordLSTM trained on PTB at different levels of gradient sparsity and temporal sparsity. The total sparsity, defined as the product of temporal and gradient sparsity remains constant along the diagonals of the matrix. We observe that different forms of sparsity perform best during different stages of training. Phrased differently, this means that there is not one optimal sparsity setup, but rather sparsity needs to be adapted to the current training phase to achieve optimal compression.

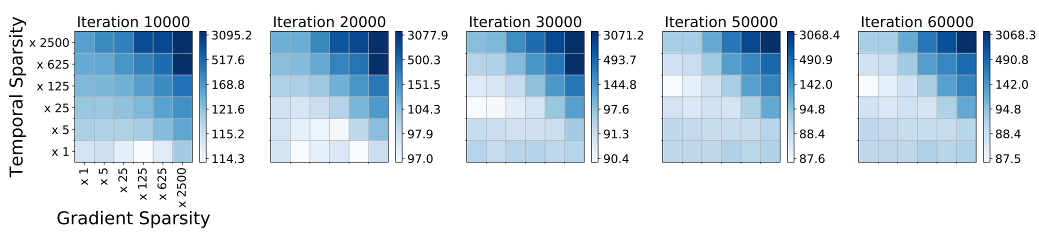


Figure 3: Perplexity for different levels of gradient sparsity and temporal sparsity at different stages of training. WordLSTM trained on PTB.