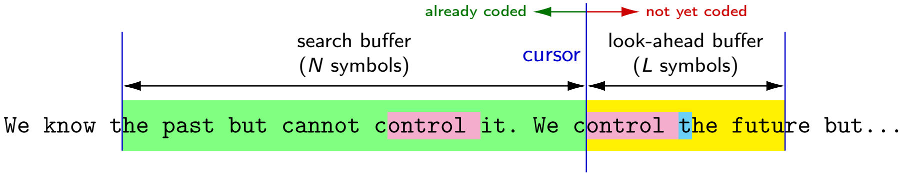
Dictionary-based Coding



Last Lecture: Predictive Lossless Coding

Predictive Lossless Coding

- Simple and effective way to exploit dependencies between neighboring symbols / samples
- Optimal predictor: Conditional mean (requires storage of large tables)

Affine and Linear Prediction

- Simple structure, low-complex implementation possible
- Optimal prediction parameters are given by solution of Yule-Walker equations
- Works very well for real signals (e.g., audio, images, ...)

Efficient Lossless Coding for Real-World Signals

- Affine/linear prediction (often: block-adaptive choice of prediction parameters)
- Entropy coding of prediction errors (e.g., arithmetic coding)
  - Using marginal pmf often already yields good results
  - Can be improved by using conditional pmfs (with simple conditions)

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Coding of Text Files

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- Affine prediction does not work (requires linear dependencies)
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➡ Including "phrases" would further increase coding efficiency

Lempel-Ziv Coding

Universal Algorithms for Lossless Data Compression

- Based on the work of Abraham Lempel and Jacob Ziv
- Basic idea: Construct dictionary during encoding and decoding

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Two Basic Variants

- **LZ77**: Based on [Ziv, Lempel, "A Universal algorithm for sequential data compresion", 1977]
  - ➡ Lempel-Ziv-Storer-Szymanski (LSZZ)
  - ➡ DEFLATE used in **ZIP**, **gzip**, **PNG**, **TIFF**, **PDF**, **OpenDocument**, ...
  - ➡ Lempel-Ziv-Markov Chain Algorithm (LZMA) used in **7zip**, **xv**, **lzip**
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- **LZ78**: Based on [Ziv, Lempel, "Compression of individual sequences via variable-rate coding", 1978]
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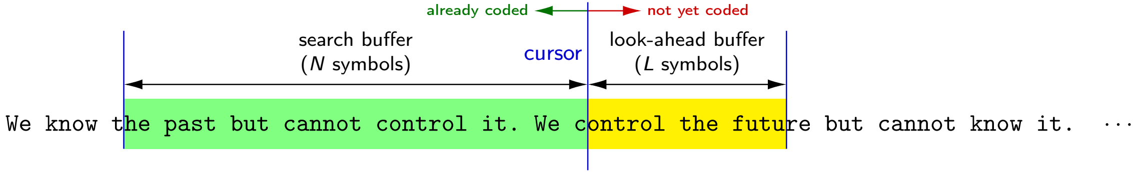
The Lempel-Ziv 1977 Algorithm (LZ77)



Basic Idea of the LZ77 Algorithm

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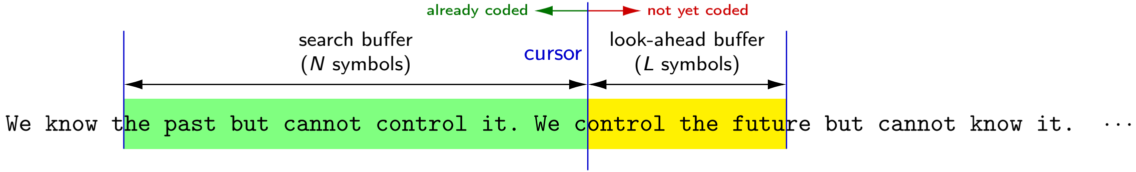

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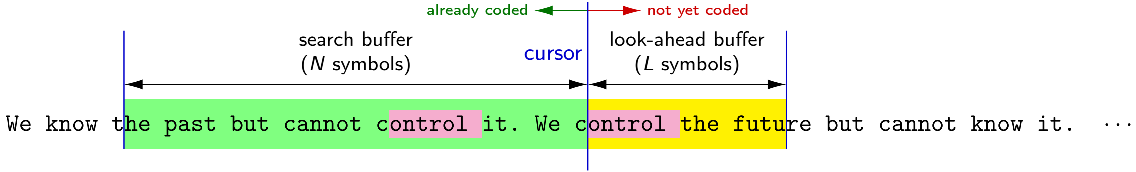

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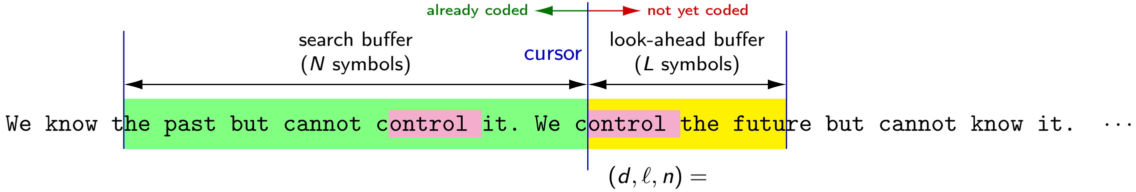

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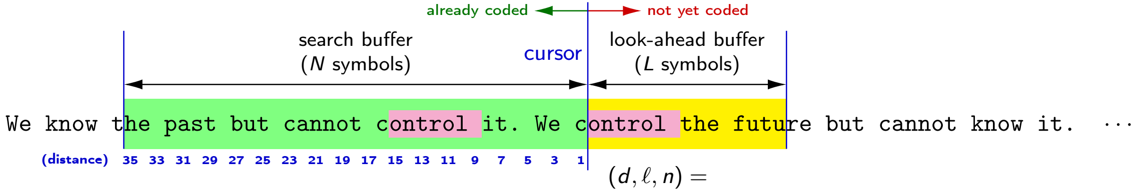

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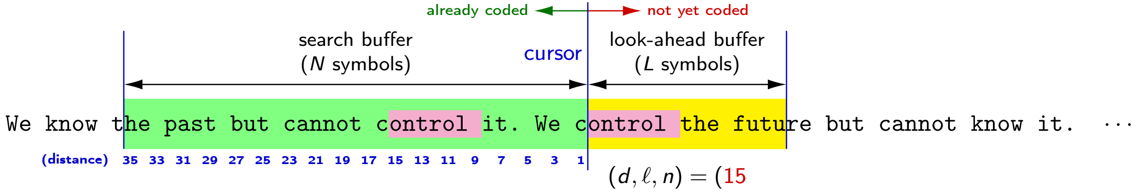

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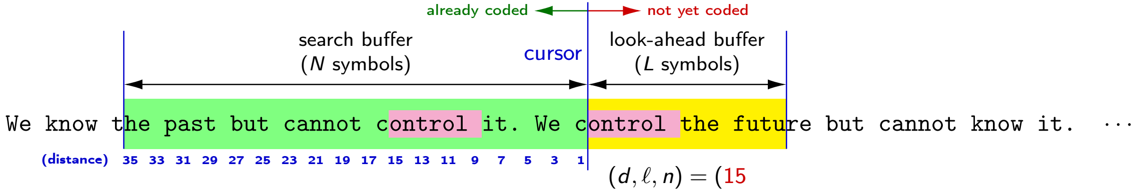

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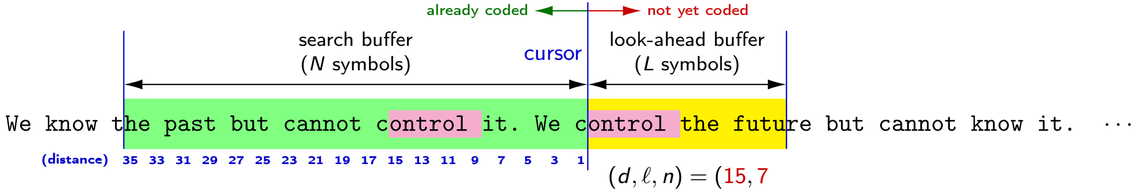

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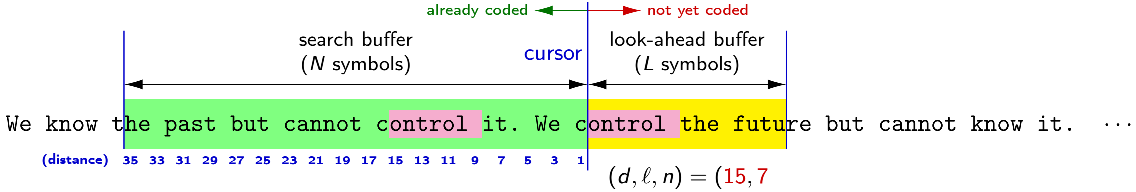

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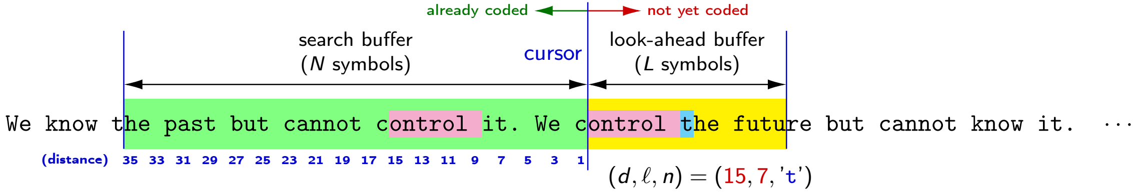

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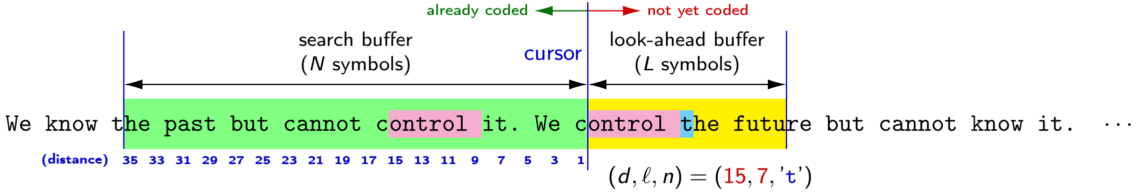

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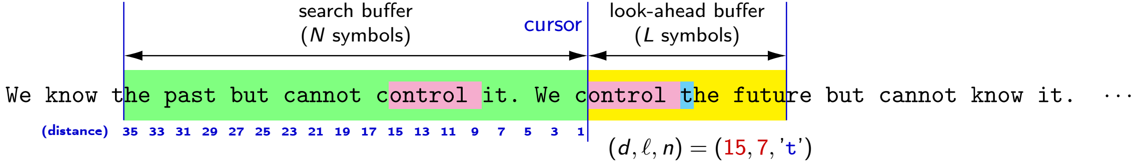
- If no match is found, then $(1, 0, n)$ is coded (with n being the next symbol after the cursor)

Simplest Version: LZ77 Algorithm with Fixed-Length Coding



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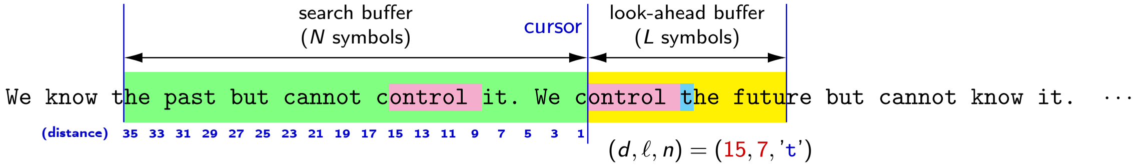

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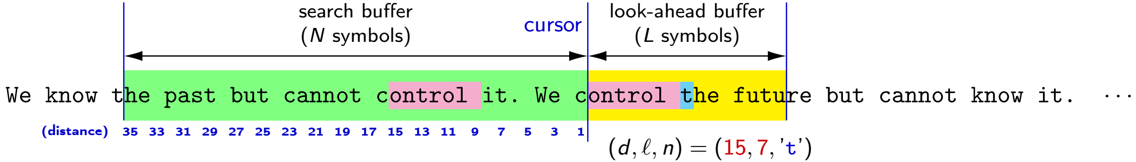

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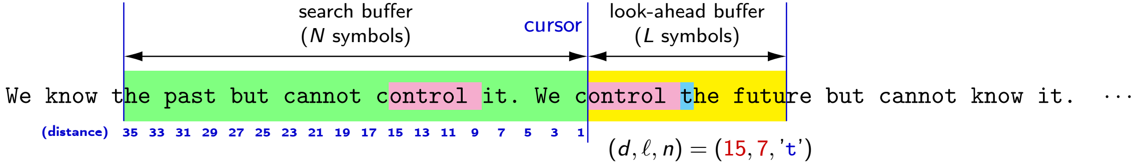

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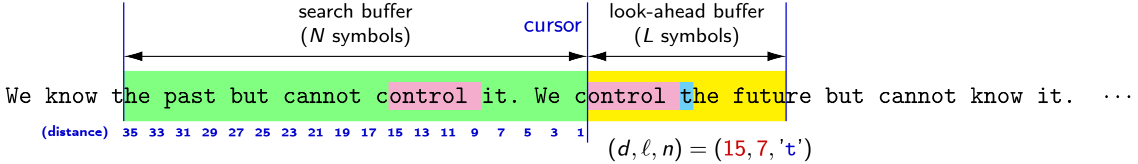

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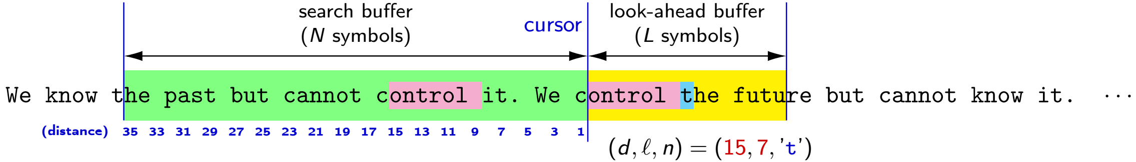

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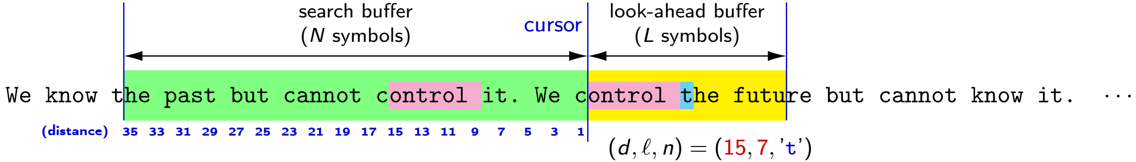

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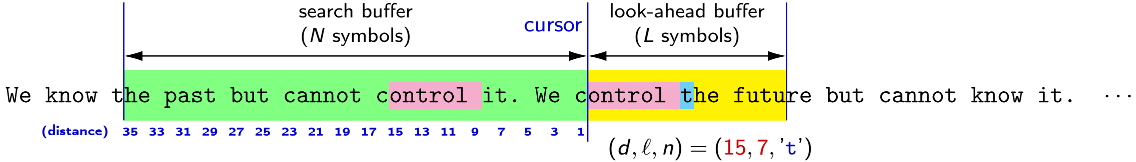

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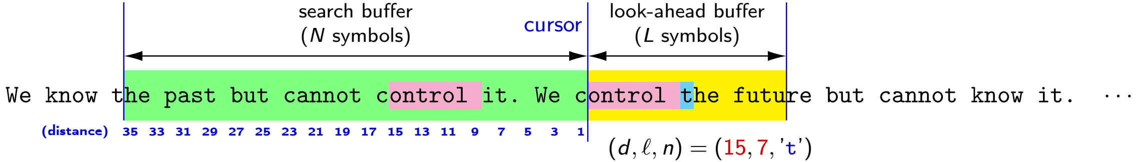

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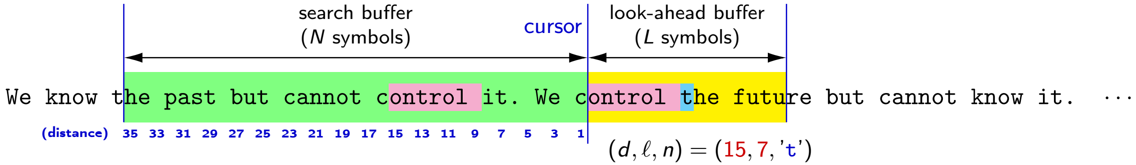

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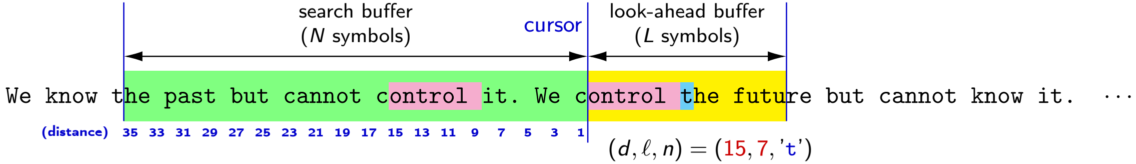

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➡ The sizes of both the preview and the look-ahead buffer should be integer powers of two !

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Message: `Miss␣Mississippi`

original message:

- 16 characters (8 bits per symbols)

➡ 128 bits (16×8 bits)

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Miss␣	Miss	

original message:

- 16 characters (8 bits per symbols)

➡ 128 bits $(16 \times 8\,\text{bits})$

LZ77 configuration:

- search buffer of $N = 8$ symbols
- look-ahead buffer of $L = 4$ symbols

Toy Example: LZ77 Encoding

Message: `Miss␣Mississippi`

search buffer	look-ahead buffer	(d, ℓ, n)
	`Miss`	(1 , 0 , M)
`M`	`iss␣`	(1 , 0 , i)
`Mi`	`ss␣M`	(1 , 0 , s)
`Mis`	`s␣Mi`	(1 , 1 , ␣)
`Miss␣`	`Miss`	(5 , 3

original message:

- 16 characters (8 bits per symbols)

→ 128 bits $(16 \times 8 \text{ bits})$

LZ77 configuration:

- search buffer of $N = 8$ symbols
- look-ahead buffer of $L = 4$ symbols

Toy Example: LZ77 Encoding

Message: Miss␣Mississippi

search buffer	look-ahead buffer	(d , ℓ , n)
	Miss	(1 , 0 , M)
M	iss␣	(1 , 0 , i)
Mi	ss␣M	(1 , 0 , s)
Mis	s␣Mi	(1 , 1 , ␣)
Miss␣	Miss	(5 , 3 , s)

original message:

- 16 characters (8 bits per symbols)

➜ 128 bits (16×8 bits)

LZ77 configuration:

- search buffer of $N = 8$ symbols
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Toy Example: LZ77 Encoding

Message: Miss␣Mississippi

search buffer	look-ahead buffer	(d, ℓ, n)
	Miss	(1 , 0 , M)
M	iss␣	(1 , 0 , i)
Mi	ss␣M	(1 , 0 , s)
Mis	s␣Mi	(1 , 1 , ␣)
Miss␣	Miss	(5 , 3 , s)
iss␣Miss	issi	

original message:

- 16 characters (8 bits per symbols)
- ➜ 128 bits $(16 \times 8\,\text{bits})$

LZ77 configuration:

- search buffer of $N = 8$ symbols
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Toy Example: LZ77 Encoding

Message: Miss␣Mississippi

search buffer	look-ahead buffer	(d, ℓ, n)
	Miss	(1 , 0 , M)
M	iss␣	(1 , 0 , i)
Mi	ss␣M	(1 , 0 , s)
Mis	s␣Mi	(1 , 1 , ␣)
Miss␣	Miss	(5 , 3 , s)
iss␣Miss	issi	(3 , 3

original message:

- 16 characters (8 bits per symbols)
- ➡ 128 bits (16 × 8 bits)

LZ77 configuration:

- search buffer of $N = 8$ symbols
- look-ahead buffer of $L = 4$ symbols

Toy Example: LZ77 Encoding

Message: Miss␣Mississippi

search buffer	look-ahead buffer	(d, ℓ, n)
	Miss	(1, 0, M)
M	iss␣	(1, 0, i)
Mi	ss␣M	(1, 0, s)
Mis	s␣Mi	(1, 1, ␣)
Miss␣	Miss	(5, 3, s)
iss␣Miss	issi	(3, 3, i)

original message:

- 16 characters (8 bits per symbols)
- ➔ 128 bits (16 × 8 bits)

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Message: Miss␣Mississippi

search buffer	look-ahead buffer	(d , ℓ , n)
	Miss	(1 , 0 , M)
M	iss␣	(1 , 0 , i)
Mi	ss␣M	(1 , 0 , s)
Mis	s␣Mi	(1 , 1 , ␣)
Miss␣	Miss	(5 , 3 , s)
iss␣Miss	issi	(3 , 3 , i)
Mississi	ppi	

original message:

- 16 characters (8 bits per symbols)
- ➜ 128 bits (16×8 bits)

LZ77 configuration:

- search buffer of $N = 8$ symbols
- look-ahead buffer of $L = 4$ symbols

Toy Example: LZ77 Encoding

Message: Miss␣Mississippi

search buffer	look-ahead buffer	(d, ℓ, n)
	Miss	(1 , 0 , M)
M	iss␣	(1 , 0 , i)
Mi	ss␣M	(1 , 0 , s)
Mis	s␣Mi	(1 , 1 , ␣)
Miss␣	Miss	(5 , 3 , s)
iss␣Miss	issi	(3 , 3 , i)
Mississi	ppi	(1 , 0

original message:

- 16 characters (8 bits per symbols)
- → 128 bits (16×8 bits)

LZ77 configuration:

- search buffer of $N = 8$ symbols
- look-ahead buffer of $L = 4$ symbols

Toy Example: LZ77 Encoding

Message: `Miss␣Mississippi`

search buffer	look-ahead buffer	(d , ℓ , n)
	Miss	(1 , 0 , M)
M	iss␣	(1 , 0 , i)
Mi	ss␣M	(1 , 0 , s)
Mis	s␣Mi	(1 , 1 , ␣)
Miss␣	Miss	(5 , 3 , s)
iss␣Miss	issi	(3 , 3 , i)
Mississi	ppi	(1 , 0 , p)

original message:

- 16 characters (8 bits per symbols)
- ➡ 128 bits (16×8 bits)

LZ77 configuration:

- search buffer of $N = 8$ symbols
- look-ahead buffer of $L = 4$ symbols

Toy Example: LZ77 Encoding

Message: Miss␣Mississippi

search buffer	look-ahead buffer	(d, ℓ, n)
	Miss	(1, 0, M)
M	iss␣	(1, 0, i)
Mi	ss␣M	(1, 0, s)
Mis	s␣Mi	(1, 1, ␣)
Miss␣	Miss	(5, 3, s)
iss␣Miss	issi	(3, 3, i)
Mississi	ppi	(1, 0, p)
ississip	pi	

original message:

- 16 characters (8 bits per symbols)
- ➔ 128 bits $(16 \times 8\,\text{bits})$

LZ77 configuration:

- search buffer of $N = 8$ symbols
- look-ahead buffer of $L = 4$ symbols

Toy Example: LZ77 Encoding

Message: Miss␣Mississippi

search buffer	look-ahead buffer	(d, ℓ, n)
	Miss	(1 , 0 , M)
M	iss␣	(1 , 0 , i)
Mi	ss␣M	(1 , 0 , s)
Mis	s␣Mi	(1 , 1 , ␣)
Miss␣	Miss	(5 , 3 , s)
iss␣Miss	issi	(3 , 3 , i)
Mississi	ppi	(1 , 0 , p)
ississip	pi	(1 , 1

original message:

- 16 characters (8 bits per symbols)
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LZ77 configuration:

- search buffer of $N = 8$ symbols
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Toy Example: LZ77 Encoding

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search buffer	look-ahead buffer	(d, ℓ, n)
	Miss	(1 , 0 , M)
M	iss␣	(1 , 0 , i)
Mi	ss␣M	(1 , 0 , s)
Mis	s␣Mi	(1 , 1 , ␣)
Miss␣	Miss	(5 , 3 , s)
iss␣Miss	issi	(3 , 3 , i)
Mississi	ppi	(1 , 0 , p)
ississip	pi	(1 , 1 , i)

original message:

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	Miss	(1 , 0 , M)
M	iss␣	(1 , 0 , i)
Mi	ss␣M	(1 , 0 , s)
Mis	s␣Mi	(1 , 1 , ␣)
Miss␣	Miss	(5 , 3 , s)
iss␣Miss	issi	(3 , 3 , i)
Mississi	ppi	(1 , 0 , p)
ississip	pi	(1 , 1 , i)

original message:

- 16 characters (8 bits per symbols)
- ➔ 128 bits (16×8 bits)

LZ77 configuration:

- search buffer of $N = 8$ symbols
- look-ahead buffer of $L = 4$ symbols

coded representation (fixed-length):

- ➔ 8 triples (d, ℓ, n)
- 13 bits per triple ($3 + 2 + 8$ bits)
- ➔ 104 bits (19% bit savings)

Toy Example: LZ77 Decoding

Coded representation: $(1, 0, \texttt{M})$ $(1, 0, \texttt{i})$ $(1, 0, \texttt{s})$ $(1, 1, \textvisiblespace)$ $(5, 3, \texttt{s})$ $(3, 3, \texttt{i})$ $(1, 0, \texttt{p})$ $(1, 1, \texttt{i})$

Decode message:

Toy Example: LZ77 Decoding

Coded representation: (1, 0, M) (1, 0, i) (1, 0, s) (1, 1, ␣) (5, 3, s) (3, 3, i) (1, 0, p) (1, 1, i)

Decode message:

search buffer	(d , ℓ , n)	decoded phrase
	(1 , 0 , M)	

Toy Example: LZ77 Decoding

Coded representation: (1, 0, M) (1, 0, i) (1, 0, s) (1, 1, ␣) (5, 3, s) (3, 3, i) (1, 0, p) (1, 1, i)

Decode message:

search buffer	(d , ℓ , n)	decoded phrase
	(1 , 0 , M)	M

Toy Example: LZ77 Decoding

Coded representation: $(1, 0, \texttt{M})$ $(1, 0, \texttt{i})$ $(1, 0, \texttt{s})$ $(1, 1, \texttt{␣})$ $(5, 3, \texttt{s})$ $(3, 3, \texttt{i})$ $(1, 0, \texttt{p})$ $(1, 1, \texttt{i})$

Decode message: M

search buffer	(d, ℓ, n)	decoded phrase
	$(1, 0, \texttt{M})$	M
M	$(1, 0, \texttt{i})$	

Toy Example: LZ77 Decoding

Coded representation: (1, 0, M) (1, 0, i) (1, 0, s) (1, 1, ␣) (5, 3, s) (3, 3, i) (1, 0, p) (1, 1, i)

Decode message: M

search buffer	(d, ℓ, n)	decoded phrase
	(1, 0, M)	M
M	(1, 0, i)	i

Toy Example: LZ77 Decoding

Coded representation: (1, 0, M) (1, 0, i) (1, 0, s) (1, 1, ␣) (5, 3, s) (3, 3, i) (1, 0, p) (1, 1, i)

Decode message: Mi

search buffer	(d , ℓ , n)	decoded phrase
	(1 , 0 , M)	M
M	(1 , 0 , i)	i
Mi	(1 , 0 , s)	

Toy Example: LZ77 Decoding

Coded representation: (1, 0, M) (1, 0, i) (1, 0, s) (1, 1, ␣) (5, 3, s) (3, 3, i) (1, 0, p) (1, 1, i)

Decode message: Mi

search buffer	(d, ℓ, n)	decoded phrase
	(1, 0, M)	M
M	(1, 0, i)	i
Mi	(1, 0, s)	s

Toy Example: LZ77 Decoding

Coded representation: (1, 0, M) (1, 0, i) (1, 0, s) (1, 1, ␣) (5, 3, s) (3, 3, i) (1, 0, p) (1, 1, i)

Decode message: Mis

search buffer	(d, ℓ, n)	decoded phrase
	(1 , 0 , M)	M
M	(1 , 0 , i)	i
Mi	(1 , 0 , s)	s
Mis	(1 , 1 , ␣)	

Toy Example: LZ77 Decoding

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Decode message: Mis

search buffer	(d , ℓ , n)	decoded phrase
	(1 , 0 , M)	M
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Toy Example: LZ77 Decoding

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Decode message: Mis

search buffer	(d, ℓ, n)	decoded phrase
	(1, 0, M)	M
M	(1, 0, i)	i
Mi	(1, 0, s)	s
Mis	(1, 1, ␣)	s␣

Toy Example: LZ77 Decoding

Coded representation: (1, 0, M) (1, 0, i) (1, 0, s) (1, 1, ␣) (5, 3, s) (3, 3, i) (1, 0, p) (1, 1, i)

Decode message: Miss␣

search buffer	(d , ℓ , n)	decoded phrase
	(1 , 0 , M)	M
M	(1 , 0 , i)	i
Mi	(1 , 0 , s)	s
Mis	(1 , 1 , ␣)	s␣
Miss␣	(5 , 3 , s)	

Toy Example: LZ77 Decoding

Coded representation: (1, 0, M) (1, 0, i) (1, 0, s) (1, 1, ␣) (5, 3, s) (3, 3, i) (1, 0, p) (1, 1, i)

Decode message: Miss␣

search buffer	(d , ℓ , n)	decoded phrase
	(1 , 0 , M)	M
M	(1 , 0 , i)	i
Mi	(1 , 0 , s)	s
Mis	(1 , 1 , ␣)	s␣
Miss␣	(5 , 3 , s)	

Toy Example: LZ77 Decoding

Coded representation: (1, 0, M) (1, 0, i) (1, 0, s) (1, 1, ␣) (5, 3, s) (3, 3, i) (1, 0, p) (1, 1, i)

Decode message: Miss␣

search buffer	(d , ℓ , n)	decoded phrase
	(1 , 0 , M)	M
M	(1 , 0 , i)	i
Mi	(1 , 0 , s)	s
Mis	(1 , 1 , ␣)	s␣
Miss␣	(5 , 3 , s)	Miss

Toy Example: LZ77 Decoding

Coded representation: (1, 0, M) (1, 0, i) (1, 0, s) (1, 1, ␣) (5, 3, s) (3, 3, i) (1, 0, p) (1, 1, i)

Decode message: Miss␣Miss

search buffer	(d , ℓ , n)	decoded phrase
	(1 , 0 , M)	M
M	(1 , 0 , i)	i
Mi	(1 , 0 , s)	s
Mis	(1 , 1 , ␣)	s␣
Miss␣	(5 , 3 , s)	Miss
iss␣Miss	(3 , 3 , i)	

Toy Example: LZ77 Decoding

Coded representation: (1, 0, M) (1, 0, i) (1, 0, s) (1, 1, ␣) (5, 3, s) (3, 3, i) (1, 0, p) (1, 1, i)

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search buffer	(d , ℓ , n)	decoded phrase
	(1 , 0 , M)	M
M	(1 , 0 , i)	i
Mi	(1 , 0 , s)	s
Mis	(1 , 1 , ␣)	s␣
Miss␣	(5 , 3 , s)	Miss
iss␣Miss	(3 , 3 , i)	

Toy Example: LZ77 Decoding

Coded representation: (1, 0, M) (1, 0, i) (1, 0, s) (1, 1, ␣) (5, 3, s) (3, 3, i) (1, 0, p) (1, 1, i)

Decode message: Miss␣Miss

search buffer	(d , ℓ , n)	decoded phrase
	(1 , 0 , M)	M
M	(1 , 0 , i)	i
Mi	(1 , 0 , s)	s
Mis	(1 , 1 , ␣)	s␣
Miss␣	(5 , 3 , s)	Miss
iss␣Miss	(3 , 3 , i)	issi

Toy Example: LZ77 Decoding

Coded representation: (1, 0, M) (1, 0, i) (1, 0, s) (1, 1, ␣) (5, 3, s) (3, 3, i) (1, 0, p) (1, 1, i)

Decode message: Miss␣Mississi

search buffer	(d, ℓ, n)	decoded phrase
	(1 , 0 , M)	M
M	(1 , 0 , i)	i
Mi	(1 , 0 , s)	s
Mis	(1 , 1 , ␣)	s␣
Miss␣	(5 , 3 , s)	Miss
iss␣Miss	(3 , 3 , i)	issi
Mississi	(1 , 0 , p)	

Toy Example: LZ77 Decoding

Coded representation: (1, 0, M) (1, 0, i) (1, 0, s) (1, 1, ␣) (5, 3, s) (3, 3, i) (1, 0, p) (1, 1, i)

Decode message: Miss␣Mississi

search buffer	(d , ℓ , n)	decoded phrase
	(1 , 0 , M)	M
M	(1 , 0 , i)	i
Mi	(1 , 0 , s)	s
Mis	(1 , 1 , ␣)	s␣
Miss␣	(5 , 3 , s)	Miss
iss␣Miss	(3 , 3 , i)	issi
Mississi	(1 , 0 , p)	p

Toy Example: LZ77 Decoding

Coded representation: (1, 0, M) (1, 0, i) (1, 0, s) (1, 1, ␣) (5, 3, s) (3, 3, i) (1, 0, p) (1, 1, i)

Decode message: Miss␣Mississip

search buffer	(d , ℓ , n)	decoded phrase
	(1 , 0 , M)	M
M	(1 , 0 , i)	i
Mi	(1 , 0 , s)	s
Mis	(1 , 1 , ␣)	s␣
Miss␣	(5 , 3 , s)	Miss
iss␣Miss	(3 , 3 , i)	issi
Mississi	(1 , 0 , p)	p
ississip	(1 , 1 , i)	

Toy Example: LZ77 Decoding

Coded representation: (1, 0, M) (1, 0, i) (1, 0, s) (1, 1, ␣) (5, 3, s) (3, 3, i) (1, 0, p) (1, 1, i)

Decode message: Miss␣Mississip

search buffer	(d , ℓ , n)	decoded phrase
	(1 , 0 , M)	M
M	(1 , 0 , i)	i
Mi	(1 , 0 , s)	s
Mis	(1 , 1 , ␣)	s␣
Miss␣	(5 , 3 , s)	Miss
iss␣Miss	(3 , 3 , i)	issi
Mississi	(1 , 0 , p)	p
ississip	(1 , 1 , i)	

Toy Example: LZ77 Decoding

Coded representation: (1, 0, M) (1, 0, i) (1, 0, s) (1, 1, ␣) (5, 3, s) (3, 3, i) (1, 0, p) (1, 1, i)

Decode message: Miss␣Mississip

search buffer	(d , ℓ , n)	decoded phrase
	(1 , 0 , M)	M
M	(1 , 0 , i)	i
Mi	(1 , 0 , s)	s
Mis	(1 , 1 , ␣)	s␣
Miss␣	(5 , 3 , s)	Miss
iss␣Miss	(3 , 3 , i)	issi
Mississi	(1 , 0 , p)	p
ississip	(1 , 1 , i)	pi

Toy Example: LZ77 Decoding

Coded representation: (1, 0, M) (1, 0, i) (1, 0, s) (1, 1, ␣) (5, 3, s) (3, 3, i) (1, 0, p) (1, 1, i)

Decode message: Miss␣Mississippi (done)

search buffer	(d , ℓ , n)	decoded phrase
	(1 , 0 , M)	M
M	(1 , 0 , i)	i
Mi	(1 , 0 , s)	s
Mis	(1 , 1 , ␣)	s␣
Miss␣	(5 , 3 , s)	Miss
iss␣Miss	(3 , 3 , i)	issi
Mississi	(1 , 0 , p)	p
ississip	(1 , 1 , i)	pi

Coding Efficiency and Complexity of LZ77

Coding Efficiency

- The LZ77 algorithm is asymptotically optimal (e.g., when using unary codes for d and ℓ)

$$N \to \infty,\, L \to \infty \qquad \Longrightarrow \qquad \bar{\ell} \to \bar{H}$$

- Proof can be found in [Cover, Thomas, "Elements of Information Theory"]
- In practice: Require really large search buffer sizes N

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- **Decoder**: Very low complexity (just copying characters)

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  - ➡ Not necessary to find the "best match" (note: shorter match can actually be more efficient)

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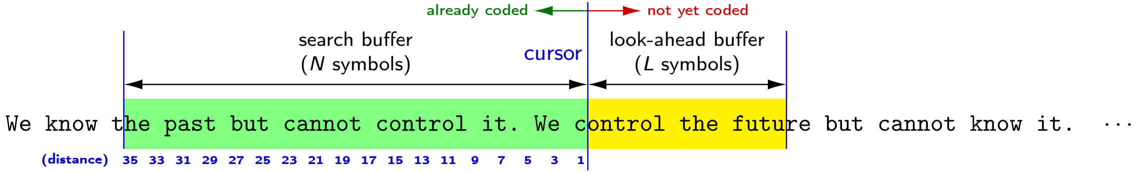
Implementation Complexity

- **Decoder**: Very low complexity (just copying characters)
- **Encoder**: Highly depends on buffer size N and actual implementation
  - ➡ Use suitable data structures such as search trees, radix trees, hash tables
  - ➡ Not necessary to find the "best match" (note: shorter match can actually be more efficient)
  - ➡ There are very efficient implementations for rather large buffer sizes (e.g., $N = 32\,768$)

LZ77 Variant: The Lempel-Ziv-Storer-Szymanski Algorithm (LZSS)



Changes relative to LZ77 Algorithm

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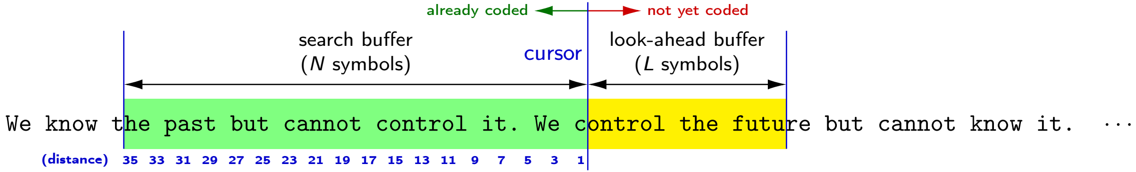

Changes relative to LZ77 Algorithm

1. At first, code a single bit b to indicate whether a match is found

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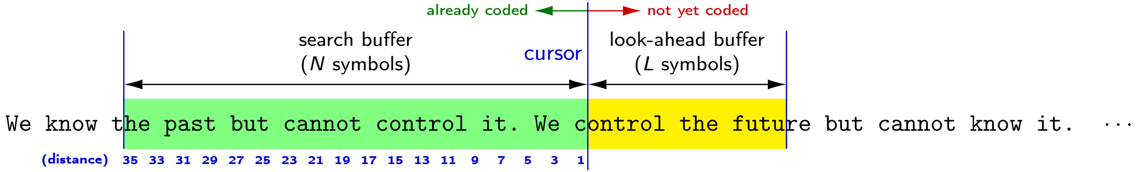

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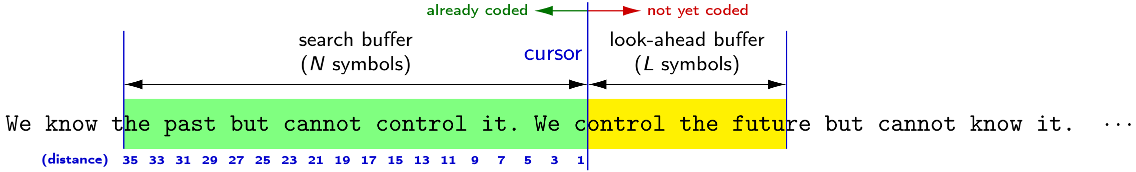
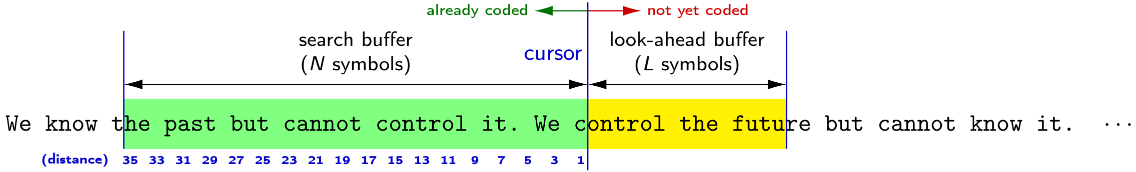


Changes relative to LZ77 Algorithm

1. At first, code a single bit b to indicate whether a match is found
2. For matches, don't transmit the following symbol

➡ **Message is coded as sequence of tuples $(b, \{d, \ell\} \mid n)$**

LZ77 Variant: The Lempel-Ziv-Storer-Szymanski Algorithm (LZSS)



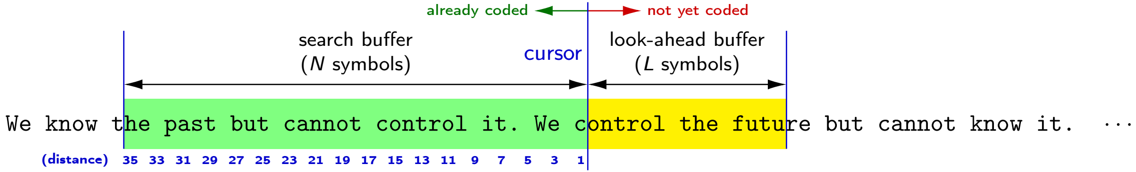
Changes relative to LZ77 Algorithm

1. At first, code a single bit b to indicate whether a match is found
2. For matches, don't transmit the following symbol

➡ **Message is coded as sequence of tuples** $(b, \{d, \ell\} \mid n)$

- The indication bit b signals whether a match is found ($b = 1 \;\rightarrow\;$ match found)

LZ77 Variant: The Lempel-Ziv-Storer-Szymanski Algorithm (LZSS)



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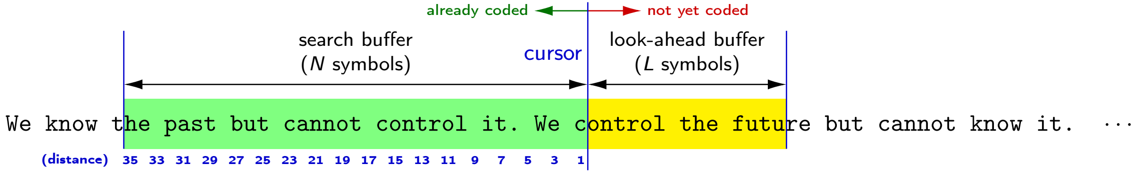

Changes relative to LZ77 Algorithm

1. At first, code a single bit b to indicate whether a match is found
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➡ **Message is coded as sequence of tuples $(b, \{d, \ell\} \mid n)$**

- The indication bit b signals whether a match is found ($b = 1 \rightarrow$ match found)
- If ($b = 0$), then code next symbol n as literal
- If ($b = 1$), then code the match as distance-length pair $\{d, \ell\}$ (with $d \in [1, N]$ and $\ell \in [1, L]$)

Toy Example: LZSS Encoding

Message: `Miss␣Mississippi`

original message:

- 16 characters (8 bits per symbols)

➡ 128 bits (16×8 bits)

Toy Example: LZSS Encoding

Message: Miss␣Mississippi

original message:

- 16 characters (8 bits per symbols)
- ➡ 128 bits $(16 \times 8\,\text{bits})$

LZSS configuration:

- search buffer of $N = 8$ symbols
- look-ahead buffer of $L = 4$ symbols

Toy Example: LZSS Encoding

Message: Miss␣Mississippi

search buffer	look-ahead	$(\, b, \{d, \ell\} \mid n \,)$
	Miss	

original message:

- 16 characters (8 bits per symbols)
- ➡ 128 bits $(16 \times 8\,\text{bits})$

LZSS configuration:

- search buffer of $N = 8$ symbols
- look-ahead buffer of $L = 4$ symbols

Toy Example: LZSS Encoding

Message: `Miss␣Mississippi`

search buffer	look-ahead	$(b, \{d, \ell\} \mid n)$
	`Miss`	(0

original message:

- 16 characters (8 bits per symbols)

➡ 128 bits $(16 \times 8\,\text{bits})$

LZSS configuration:

- search buffer of $N = 8$ symbols
- look-ahead buffer of $L = 4$ symbols

Toy Example: LZSS Encoding

Message: `Miss␣Mississippi`

search buffer	look-ahead	$(\, b, \{d, \ell\} \mid n \,)$
	Miss	$(\, 0, \texttt{M} \,)$

original message:

- 16 characters (8 bits per symbols)
- → 128 bits (16×8 bits)

LZSS configuration:

- search buffer of $N = 8$ symbols
- look-ahead buffer of $L = 4$ symbols

Toy Example: LZSS Encoding

Message: `Miss␣Mississippi`

search buffer	look-ahead	$(\, b, \{d, \ell\} \mid n \,)$
	`Miss`	$(\, 0,$ `M` $)$
`M`	`iss␣`	

original message:

- 16 characters (8 bits per symbols)
- ➡ 128 bits (16×8 bits)

LZSS configuration:

- search buffer of $N = 8$ symbols
- look-ahead buffer of $L = 4$ symbols

Toy Example: LZSS Encoding

Message: Miss␣Mississippi

search buffer	look-ahead	$(\,b, \{d, \ell\} \mid n\,)$
	Miss	(0, M)
M	iss␣	(0

original message:

- 16 characters (8 bits per symbols)
- ➜ 128 bits (16×8 bits)

LZSS configuration:

- search buffer of $N = 8$ symbols
- look-ahead buffer of $L = 4$ symbols

Toy Example: LZSS Encoding

Message: Miss␣Mississippi

search buffer	look-ahead	$(\, b, \{d, \ell\} \mid n \,)$
	Miss	(0, M)
M	iss␣	(0, i)

original message:

- 16 characters (8 bits per symbols)
- ➡ 128 bits $(16 \times 8\,\text{bits})$

LZSS configuration:

- search buffer of $N = 8$ symbols
- look-ahead buffer of $L = 4$ symbols

Toy Example: LZSS Encoding

Message: Miss␣Mississippi

search buffer	look-ahead	$(\, b, \{d, \ell\} \mid n \,)$
	Miss	(0, M)
M	iss␣	(0, i)
Mi	ss␣M	

original message:

- 16 characters (8 bits per symbols)
- ➡ 128 bits (16×8 bits)

LZSS configuration:

- search buffer of $N = 8$ symbols
- look-ahead buffer of $L = 4$ symbols

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search buffer	look-ahead	$(\, b, \{d, \ell\} \mid n \,)$
	Miss	(0, M)
M	iss␣	(0, i)
Mi	ss␣M	(0

original message:

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search buffer	look-ahead	$(\,b, \{d, \ell\} \mid n\,)$
	Miss	(0, M)
M	iss␣	(0, i)
Mi	ss␣M	(0, s)

original message:

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Toy Example: LZSS Encoding

Message: Miss␣Mississippi

search buffer	look-ahead	$(b, \{d, \ell\} \mid n)$
	Miss	$(0, \texttt{M})$
M	iss␣	$(0, \texttt{i})$
Mi	ss␣M	$(0, \texttt{s})$
Mis	s␣Mi	

original message:

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Toy Example: LZSS Encoding

Message: Miss␣Mississippi

search buffer	look-ahead	$(\, b, \{d, \ell\} \mid n \,)$
	Miss	(0, M)
M	iss␣	(0, i)
Mi	ss␣M	(0, s)
Mis	s␣Mi	(1

original message:

- 16 characters (8 bits per symbols)
- ➡ 128 bits $(16 \times 8\,\text{bits})$

LZSS configuration:

- search buffer of $N = 8$ symbols
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Message: Miss␣Mississippi

search buffer	look-ahead	$(\, b, \{d, \ell\} \mid n \,)$
	Miss	(0, M)
M	iss␣	(0, i)
Mi	ss␣M	(0, s)
Mis	s␣Mi	(1, 1, 1)

original message:

- 16 characters (8 bits per symbols)
- ➡ 128 bits $(16 \times 8\,\text{bits})$

LZSS configuration:

- search buffer of $N = 8$ symbols
- look-ahead buffer of $L = 4$ symbols

Toy Example: LZSS Encoding

Message: Miss␣Mississippi

search buffer	look-ahead	$(\, b, \{d, \ell\} \mid n \,)$
	Miss	$(\, 0, \text{M} \,)$
M	iss␣	$(\, 0, \text{i} \,)$
Mi	ss␣M	$(\, 0, \text{s} \,)$
Mis	s␣Mi	$(\, 1, 1, 1 \,)$
Miss	␣Mis	

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- search buffer of $N = 8$ symbols
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Toy Example: LZSS Encoding

Message: `Miss␣Mississippi`

search buffer	look-ahead	$(\, b, \{d, \ell\} \mid n \,)$
	`Miss`	$(\, 0, \texttt{M} \,)$
`M`	`iss␣`	$(\, 0, \texttt{i} \,)$
`Mi`	`ss␣M`	$(\, 0, \texttt{s} \,)$
`Mis`	`s␣Mi`	$(\, 1, 1, 1 \,)$
`Miss`	`␣Mis`	$(\, 0$

original message:

- 16 characters (8 bits per symbols)
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LZSS configuration:

- search buffer of $N = 8$ symbols
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Toy Example: LZSS Encoding

Message: Miss␣Mississippi

search buffer	look-ahead	$(\, b, \{d, \ell\} \mid n \,)$
	Miss	(0, M)
M	iss␣	(0, i)
Mi	ss␣M	(0, s)
Mis	s␣Mi	(1, 1, 1)
Miss	␣Mis	(0, ␣)

original message:

- 16 characters (8 bits per symbols)
- ➡ 128 bits (16×8 bits)

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- search buffer of $N = 8$ symbols
- look-ahead buffer of $L = 4$ symbols

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	`Miss`	$(\,0, \texttt{M}\,)$
`M`	`iss␣`	$(\,0, \texttt{i}\,)$
`Mi`	`ss␣M`	$(\,0, \texttt{s}\,)$
`Mis`	`s␣Mi`	$(\,1, 1, 1\,)$
`Miss`	`␣Mis`	$(\,0, \texttt{␣}\,)$
`Miss␣`	`Miss`	

original message:

- 16 characters (8 bits per symbols)

➡ 128 bits $(16 \times 8\,\text{bits})$

LZSS configuration:

- search buffer of $N = 8$ symbols
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search buffer	look-ahead	$(\, b, \{d, \ell\} \mid n \,)$
	Miss	(0, M)
M	iss␣	(0, i)
Mi	ss␣M	(0, s)
Mis	s␣Mi	(1, 1, 1)
Miss	␣Mis	(0, ␣)
Miss␣	Miss	(1

original message:

- 16 characters (8 bits per symbols)
- ➡ 128 bits (16×8 bits)

LZSS configuration:

- search buffer of $N = 8$ symbols
- look-ahead buffer of $L = 4$ symbols

Toy Example: LZSS Encoding

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search buffer	look-ahead	$(\, b, \{d, \ell\} \mid n \,)$
	Miss	(0, M)
M	iss␣	(0, i)
Mi	ss␣M	(0, s)
Mis	s␣Mi	(1, 1, 1)
Miss	␣Mis	(0, ␣)
Miss␣	Miss	(1, 5, 4)

original message:

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	Miss	(0, M)
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Mi	ss␣M	(0, s)
Mis	s␣Mi	(1, 1, 1)
Miss	␣Mis	(0, ␣)
Miss␣	Miss	(1, 5, 4)
iss␣Miss	issi	

original message:

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LZSS configuration:

- search buffer of $N = 8$ symbols
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Message: Miss␣Mississippi

search buffer	look-ahead	$(\, b, \{d, \ell\} \mid n \,)$
	Miss	(0, M)
M	iss␣	(0, i)
Mi	ss␣M	(0, s)
Mis	s␣Mi	(1, 1, 1)
Miss	␣Mis	(0, ␣)
Miss␣	Miss	(1, 5, 4)
iss␣Miss	issi	(1

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	Miss	(0, M)
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Mi	ss␣M	(0, s)
Mis	s␣Mi	(1, 1, 1)
Miss	␣Mis	(0, ␣)
Miss␣	Miss	(1, 5, 4)
iss␣Miss	issi	(1, 3, 4)

original message:

- 16 characters (8 bits per symbols)

➡ 128 bits $(16 \times 8\,\text{bits})$

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- search buffer of $N = 8$ symbols
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	Miss	(0, M)
M	iss␣	(0, i)
Mi	ss␣M	(0, s)
Mis	s␣Mi	(1, 1, 1)
Miss	␣Mis	(0, ␣)
Miss␣	Miss	(1, 5, 4)
iss␣Miss	issi	(1, 3, 4)
Mississi	ppi	

original message:

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Mi	ss␣M	(0, s)
Mis	s␣Mi	(1, 1, 1)
Miss	␣Mis	(0, ␣)
Miss␣	Miss	(1, 5, 4)
iss␣Miss	issi	(1, 3, 4)
Mississi	ppi	(0

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Toy Example: LZSS Encoding

Message: Miss␣Mississippi

search buffer	look-ahead	$(\, b, \{d, \ell\} \mid n \,)$
	Miss	(0, M)
M	iss␣	(0, i)
Mi	ss␣M	(0, s)
Mis	s␣Mi	(1, 1, 1)
Miss	␣Mis	(0, ␣)
Miss␣	Miss	(1, 5, 4)
iss␣Miss	issi	(1, 3, 4)
Mississi	ppi	(0, p)

original message:

- 16 characters (8 bits per symbols)
- ➜ 128 bits $(16 \times 8\,\text{bits})$

LZSS configuration:

- search buffer of $N = 8$ symbols
- look-ahead buffer of $L = 4$ symbols

Toy Example: LZSS Encoding

Message: `Miss␣Mississippi`

search buffer	look-ahead	$(\,b, \{d, \ell\} \mid n\,)$
	`Miss`	(0, `M`)
`M`	`iss␣`	(0, `i`)
`Mi`	`ss␣M`	(0, `s`)
`Mis`	`s␣Mi`	(1, 1, 1)
`Miss`	`␣Mis`	(0, `␣`)
`Miss␣`	`Miss`	(1, 5, 4)
`iss␣Miss`	`issi`	(1, 3, 4)
`Mississi`	`ppi`	(0, `p`)
`ississip`	`pi`	

original message:

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LZSS configuration:

- search buffer of $N = 8$ symbols
- look-ahead buffer of $L = 4$ symbols

Toy Example: LZSS Encoding

Message: Miss␣Mississippi

search buffer	look-ahead	$(b, \{d, \ell\} \mid n)$
	Miss	(0, M)
M	iss␣	(0, i)
Mi	ss␣M	(0, s)
Mis	s␣Mi	(1, 1, 1)
Miss	␣Mis	(0, ␣)
Miss␣	Miss	(1, 5, 4)
iss␣Miss	issi	(1, 3, 4)
Mississi	ppi	(0, p)
ississip	pi	(1

original message:

- 16 characters (8 bits per symbols)

➜ 128 bits (16×8 bits)

LZSS configuration:

- search buffer of $N = 8$ symbols
- look-ahead buffer of $L = 4$ symbols

Toy Example: LZSS Encoding

Message: Miss␣Mississippi

search buffer	look-ahead	$(b, \{d, \ell\} \mid n)$
	Miss	(0, M)
M	iss␣	(0, i)
Mi	ss␣M	(0, s)
Mis	s␣Mi	(1, 1, 1)
Miss	␣Mis	(0, ␣)
Miss␣	Miss	(1, 5, 4)
iss␣Miss	issi	(1, 3, 4)
Mississi	ppi	(0, p)
ississip	pi	(1, 1, 1)

original message:

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- search buffer of $N = 8$ symbols
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Toy Example: LZSS Encoding

Message: `Miss␣Mississippi`

search buffer	look-ahead	$(\, b, \{d, \ell\} \mid n \,)$
	Miss	(0, M)
M	iss␣	(0, i)
Mi	ss␣M	(0, s)
Mis	s␣Mi	(1, 1, 1)
Miss	␣Mis	(0, ␣)
Miss␣	Miss	(1, 5, 4)
iss␣Miss	issi	(1, 3, 4)
Mississi	ppi	(0, p)
ississip	pi	(1, 1, 1)
ssissipp	i	

original message:

- 16 characters (8 bits per symbols)
- ➜ 128 bits (16×8 bits)

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Toy Example: LZSS Encoding

Message: Miss␣Mississippi

search buffer	look-ahead	$(\, b, \{d, \ell\} \mid n \,)$
	Miss	(0, M)
M	iss␣	(0, i)
Mi	ss␣M	(0, s)
Mis	s␣Mi	(1, 1, 1)
Miss	␣Mis	(0, ␣)
Miss␣	Miss	(1, 5, 4)
iss␣Miss	issi	(1, 3, 4)
Mississi	ppi	(0, p)
ississip	pi	(1, 1, 1)
ssissipp	i	(1

original message:

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LZSS configuration:

- search buffer of $N = 8$ symbols
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Toy Example: LZSS Encoding

Message: `Miss␣Mississippi`

search buffer	look-ahead	$(\, b, \{d, \ell\} \mid n \,)$
	`Miss`	$(\, 0, \texttt{M} \,)$
`M`	`iss␣`	$(\, 0, \texttt{i} \,)$
`Mi`	`ss␣M`	$(\, 0, \texttt{s} \,)$
`Mis`	`s␣Mi`	$(\, 1, 1, 1 \,)$
`Miss`	`␣Mis`	$(\, 0, \texttt{␣} \,)$
`Miss␣`	`Miss`	$(\, 1, 5, 4 \,)$
`iss␣Miss`	`issi`	$(\, 1, 3, 4 \,)$
`Mississi`	`ppi`	$(\, 0, \texttt{p} \,)$
`ississip`	`pi`	$(\, 1, 1, 1 \,)$
`ssissipp`	`i`	$(\, 1, 3, 1 \,)$

original message:

- 16 characters (8 bits per symbols)
- ➜ 128 bits (16×8 bits)

LZSS configuration:

- search buffer of $N = 8$ symbols
- look-ahead buffer of $L = 4$ symbols

Toy Example: LZSS Encoding

Message: Miss␣Missississippi

search buffer	look-ahead	$(\,b, \{d, \ell\} \mid n\,)$
	Miss	(0, M)
M	iss␣	(0, i)
Mi	ss␣M	(0, s)
Mis	s␣Mi	(1, 1, 1)
Miss	␣Mis	(0, ␣)
Miss␣	Miss	(1, 5, 4)
iss␣Miss	issi	(1, 3, 4)
Mississi	ppi	(0, p)
ississip	pi	(1, 1, 1)
ssissipp	i	(1, 3, 1)

original message:

- 16 characters (8 bits per symbols)
- ➜ 128 bits $(16 \times 8\,\text{bits})$

LZSS configuration:

- search buffer of $N = 8$ symbols
- look-ahead buffer of $L = 4$ symbols

coded representation (fixed-length):

- ➜ 5 literals $(5 \times 9$ bits)
- ➜ 5 matches $(5 \times 6$ bits)
- ➜ 75 bits (41% bit savings)

Toy Example: LZSS Decoding

Coded representation: $(0, \texttt{M})$ $(0, \texttt{i})$ $(0, \texttt{s})$ $(1, 1, 1)$ $(0, \texttt{␣})$ $(1, 5, 4)$ $(1, 3, 4)$ $(0, \texttt{p})$ $(1, 1, 1)$ $(1, 3, 1)$

Decode message:

Toy Example: LZSS Decoding

Coded representation: $(0, \texttt{M})$ $(0, \texttt{i})$ $(0, \texttt{s})$ $(1, 1, 1)$ $(0, \texttt{␣})$ $(1, 5, 4)$ $(1, 3, 4)$ $(0, \texttt{p})$ $(1, 1, 1)$ $(1, 3, 1)$

Decode message:

search buffer	$(\, b,\, \{d,\, \ell\} \,\vert\, n \,)$	decoded phrase
	$(\, 0,\, \texttt{M} \,)$	

Toy Example: LZSS Decoding

Coded representation: $(0, \mathtt{M})$ $(0, \mathtt{i})$ $(0, \mathtt{s})$ $(1, 1, 1)$ $(0, \mathtt{␣})$ $(1, 5, 4)$ $(1, 3, 4)$ $(0, \mathtt{p})$ $(1, 1, 1)$ $(1, 3, 1)$

Decode message:

search buffer	$(\, b,\, \{d,\, \ell\} \,|\, n \,)$	decoded phrase
	$(\, 0,\, \mathtt{M} \,)$	M

Toy Example: LZSS Decoding

Coded representation: (0, M) (0, i) (0, s) (1, 1, 1) (0, ␣) (1, 5, 4) (1, 3, 4) (0, p) (1, 1, 1) (1, 3, 1)

Decode message: M

search buffer	$(\,b,\,\{d,\,\ell\}\,|\,n\,)$	decoded phrase
	(0, M)	M
M	(0, i)	

Toy Example: LZSS Decoding

Coded representation: (0, M) (0, i) (0, s) (1, 1, 1) (0, ␣) (1, 5, 4) (1, 3, 4) (0, p) (1, 1, 1) (1, 3, 1)

Decode message: M

search buffer	$(b, \{d, \ell\} \mid n)$	decoded phrase
	(0, M)	M
M	(0, i)	i

Toy Example: LZSS Decoding

Coded representation: (0, M) (0, i) (0, s) (1, 1, 1) (0, ␣) (1, 5, 4) (1, 3, 4) (0, p) (1, 1, 1) (1, 3, 1)

Decode message: Mi

search buffer	$(\, b,\ \{d,\ \ell\} \mid n\,)$	decoded phrase
	(0, M)	M
M	(0, i)	i
Mi	(0, s)	

Toy Example: LZSS Decoding

Coded representation: (0, M) (0, i) (0, s) (1, 1, 1) (0, ␣) (1, 5, 4) (1, 3, 4) (0, p) (1, 1, 1) (1, 3, 1)

Decode message: Mi

search buffer	$(\, b,\ \{d,\ \ell\} \,|\, n \,)$	decoded phrase
	(0, M)	M
M	(0, i)	i
Mi	(0, s)	s

Toy Example: LZSS Decoding

Coded representation: $(0, \texttt{M})$ $(0, \texttt{i})$ $(0, \texttt{s})$ $(1, 1, 1)$ $(0, \texttt{␣})$ $(1, 5, 4)$ $(1, 3, 4)$ $(0, \texttt{p})$ $(1, 1, 1)$ $(1, 3, 1)$

Decode message: Mis

search buffer	$(b, \{d, \ell\} \mid n)$	decoded phrase
	$(0, \texttt{M})$	M
M	$(0, \texttt{i})$	i
Mi	$(0, \texttt{s})$	s
Mis	$(1, 1, 1)$	

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Decode message: Mis

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Decode message: Mis

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Mi	(0, s)	s
Mis	(1, 1, 1)	s
Miss	(0, ␣)	␣

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Decode message: Miss␣

search buffer	$(\, b,\, \{d,\, \ell\} \,|\, n \,)$	decoded phrase
	(0, M)	M
M	(0, i)	i
Mi	(0, s)	s
Mis	(1, 1, 1)	s
Miss	(0, ␣)	␣
Miss␣	(1, 5, 4)	

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Coded representation: (0, M) (0, i) (0, s) (1, 1, 1) (0, ␣) (1, 5, 4) (1, 3, 4) (0, p) (1, 1, 1) (1, 3, 1)

Decode message: Miss␣

search buffer	(b, { d, ℓ } | n)	decoded phrase
	(0, M)	M
M	(0, i)	i
Mi	(0, s)	s
Mis	(1, 1, 1)	s
Miss	(0, ␣)	␣
Miss␣	(1, 5, 4)	

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M	(0, i)	i
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Miss	(0, ␣)	␣	
Miss␣	(1, 5, 4)	Miss	
iss␣Miss	(1, 3, 4)	issi	(note: *copy symbol by symbol*)

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Decode message: Miss␣Mississi

search buffer	(b, $\{d,\ \ell\}$ | n)	decoded phrase	
	(0, M)	M	
M	(0, i)	i	
Mi	(0, s)	s	
Mis	(1, 1, 1)	s	
Miss	(0, ␣)	␣	
Miss␣	(1, 5, 4)	Miss	
iss␣Miss	(1, 3, 4)	issi	(note: *copy symbol by symbol*)
Mississi	(0, p)		

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Decode message: Miss␣Mississi

search buffer	$(\,b,\,\{d,\,\ell\}\,\vert\,n\,)$	decoded phrase	
	(0, M)	M	
M	(0, i)	i	
Mi	(0, s)	s	
Mis	(1, 1, 1)	s	
Miss	(0, ␣)	␣	
Miss␣	(1, 5, 4)	Miss	
iss␣Miss	(1, 3, 4)	issi	(note: *copy symbol by symbol*)
Mississi	(0, p)	p	

Toy Example: LZSS Decoding

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Decode message: Miss␣Mississip

search buffer	(b, $\{d,\ \ell\}$ | n)	decoded phrase	
	(0, M)	M	
M	(0, i)	i	
Mi	(0, s)	s	
Mis	(1, 1, 1)	s	
Miss	(0, ␣)	␣	
Miss␣	(1, 5, 4)	Miss	
iss␣Miss	(1, 3, 4)	issi	(note: *copy symbol by symbol*)
Mississi	(0, p)	p	
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M	(0, i)	i	
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Miss	(0, ␣)	␣	
Miss␣	(1, 5, 4)	Miss	
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M	(0, i)	i	
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Miss	(0, ␣)	␣	
Miss␣	(1, 5, 4)	Miss	
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Decode message: Miss␣Mississipp

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	(0, M)	M	
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Decode message: Miss␣Mississippi (done)

search buffer	$(\, b,\ \{d,\ \ell\} \,\vert\, n \,)$	decoded phrase	
	(0, M)	M	
M	(0, i)	i	
Mi	(0, s)	s	
Mis	(1, 1, 1)	s	
Miss	(0, ␣)	␣	
Miss␣	(1, 5, 4)	Miss	
iss␣Miss	(1, 3, 4)	issi	(note: *copy symbol by symbol*)
Mississi	(0, p)	p	
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sissipp	(1, 3, 1)	i	

The DEFLATE Algorithm: Combining LZSS with Huffman Coding

The Concept of DEFLATE

- Pre-process message/file/symbol sequence using the **LZSS algorithm** (remove dependencies)
- Entropy coding of tuples $(b, \{d, \ell\} \mid n)$ using **Huffman coding**

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- Input as interpreted as sequence of bytes (alphabet size of 256)

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3-bit block header (at start of each block)		
1 bit	0	there are blocks that follow the current block
	1	this is the last block of the file / data stream
2 bits	00	uncompressed block (number of bytes in block is coded after block header, max. 65k)
	01	compressed block using pre-defined Huffman tables
	10	compressed block with transmitted Huffman tables **(most frequently used type)**
	11	reserved (forbidden)

The DEFLATE Format: Two Huffman Tables

Main Huffman table with 288 codewords

index n	**meaning** (additional codewords follow for $n = 257 \ldots 285$)	
0 – 255	literal with ASCII code being equal to n	
256	**end-of-block** (last symbol of a block)	
257 – 264	match with $\ell = (n - 254)$	
265 – 268	match with $\ell = 2 \cdot (n - 260) + 1 + x$	(1 extra bit for x)
269 – 272	match with $\ell = 4 \cdot (n - 265) + 3 + x$	(2 extra bits for x)
273 – 276	match with $\ell = 8 \cdot (n - 269) + 3 + x$	(3 extra bits for x)
277 – 280	match with $\ell = 16 \cdot (n - 273) + 3 + x$	(4 extra bits for x)
281 – 284	match with $\ell = 32 \cdot (n - 277) + 3 + x$	(5 extra bits for x)
285	match with $\ell = 258$	
286 – 287	reserved (forbidden codeword)	

Note 1: The values for x are coded using fixed-length codes.

Note 2: The match size must be in range $\ell = 3 \ldots 258$.

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Huffman table for distance

n	distance d	bits for z
0 – 3	$d = 1 + n$	
4	$d = 5 + z$	1
5	$d = 7 + z$	1
6	$d = 9 + z$	2
7	$d = 13 + z$	2
8	$d = 17 + z$	4
⋮	⋮	⋮
26	$d = 8\,193 + z$	12
27	$d = 12\,289 + z$	12
28	$d = 16\,385 + z$	13
29	$d = 24\,577 + z$	13
30 – 31	reserved	

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The DEFLATE Algorithm in Practice

Encoding and Decoding

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  - ➡ Simplified search for finding best matches

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- One the most used algorithms in practice
  - ➡ Archive formats: Library **zlib**, **ZIP**, **gzip**, **PKZIP**, **Zopfli**, **CAB**

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  - ➡ Cryptography: **Crypto++**
  - ➡ ...

LZ77 Variant: Lempel-Ziv-Markov Chain Algorithm (LZMA)

The Concept of LZMA

- Pre-process byte sequence using an **LZ77 variant** (similar to LZSS, but with special cases)
- Entropy coding of resulting bit sequence using a **range encoder** (adaptive binary arithmetic coding)

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- **Larger search buffer** of up to $\mathbf{N = 4\,294\,967\,296}$ (32 bit), look-ahead buffer of $\mathbf{L = 273}$

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Applications of LZMA

- Next generation file compressors

➞ 7zip, xv, lzip, ZIPX

LZMA: Mapping of Byte Sequences to Bit Sequences

Code for single byte sequence (match or literal)

0	+ (byte)	Direct encoding of next byte (no match)
10	$+ \ell + d$	Conventional match (followed by codes for length ℓ and distance d)
1100		Match of length $\ell = 1$, distance d is equal to last used distance
1101	$+ \ell$	Match of length ℓ, distance d is equal to last used distance
1110	$+ \ell$	Match of length ℓ, distance d is equal to second last used distance
11110	$+ \ell$	Match of length ℓ, distance d is equal to third last used distance
11111	$+ \ell$	Match of length ℓ, distance d is equal to fourth last used distance

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Code for length ℓ

0 + (3 bits)	Length in range $\ell = 2 \ldots 9$
10 + (3 bits)	Length in range $\ell = 10 \ldots 17$
11 + (8 bits)	Length in range $\ell = 18 \ldots 273$

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11	+ (8 bits)	Length in range $\ell = 18 \ldots 273$

Code for distance d

- 6 bits for indicating "distance slot"
- followed by 0–30 of bits (depending on slot)

LZMA: Entropy Coding of Bit Sequence after LZ77 Variant

Entropy Coding of Bit Sequences

- Context-based Adaptive Binary Arithmetic Coding (called range encoder)
- Multiple adaptive binary probability models + bypass mode (probability 0.5)
- Sophisticated context modeling: Probability model for next bit is chosen based on ...
  - type of bit, value of preceding byte, preceding bits of current byte,
  - type of preceding byte sequences, ...

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  - type of preceding byte sequences, ...

Binary Arithmetic Coding Engine

- 11 bits of precision for binary probability masses (only store p_0, since $p_1 = 2^{11} - p_0$)
- 32 bits of precision for interval width
- Probability models are updated according to

$$p_0 = \begin{cases} p_0 + \left((2^{11} - p_0) \gg 5 \right) & : \quad \text{bit} = 0 \\ p_0 - (p_0 \gg 5) & : \quad \text{bit} = 1 \end{cases}$$

The Lempel-Ziv 1978 Algorithm (LZ78)

Main Difference to LZ77

- Dictionary is not restricted to preceding N symbols
- Dictionary is constructed during encoding and decoding

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  - k : Index for best match in dictionary (or "0" if no match is found)
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  - n : Next symbol (similar to LZ77)
- After coding a tuple $\{k, n\}$, the represented phrase is added to the dictionary

Number of Bits for Dictionary Index

- Number of bits n_k for dictionary index depends in dictionary size

$$n_k = \left\lceil \log_2(1 + \text{dictionary size}) \right\rceil$$

- In practice: Dictionary is reset after it becomes too large

Toy Example: LZ78 Encoding

phrase	output	bits	dictionary

Message:

`thinking␣things␣through`

Remember: Number of bits for dictionary index k

$$n_k = \left\lceil \log_2(1 + \text{dictionary size}) \right\rceil$$

Toy Example: LZ78 Encoding

phrase	output	bits	dictionary
t	(0, t)	8	

Message:

thinking␣things␣through

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Toy Example: LZ78 Encoding

phrase	output	bits	dictionary
t	(0, t)	8	1: t

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Toy Example: LZ78 Encoding

phrase	output	bits	dictionary
t	(0, t)	8	1: t
h	(0, h)	9	

Message:

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Toy Example: LZ78 Encoding

phrase	output	bits	dictionary
t	(0, t)	8	1 : t
h	(0, h)	9	2 : h

Message:

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Toy Example: LZ78 Encoding

phrase	output	bits	dictionary
t	(0, t)	8	1: t
h	(0, h)	9	2: h
i	(0, i)	10	

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t	(0, t)	8	1 : t
h	(0, h)	9	2 : h
i	(0, i)	10	3 : i

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n	(0, n)	10	

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n	(0, n)	10	4: n

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i	(0, i)	10	3 : i
n	(0, n)	10	4 : n
k	(0, k)	11	

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n	(0, n)	10	4: n
k	(0, k)	11	5: k

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in	(3, n)	11	

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k	(0, k)	11	5 : k
in	(3, n)	11	6 : in

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g	(0, g)	11	7: g

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␣	(0, ␣)	11	8 : ␣

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in	(3, n)	11	6 : in
g	(0, g)	11	7 : g
␣	(0, ␣)	11	8 : ␣
th	(1, h)	12	

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␣	(0, ␣)	11	8 : ␣
th	(1, h)	12	9 : th
ing	(6, g)	12	10 : ing
s	(0, s)	12	

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␣	(0, ␣)	11	8 : ␣
th	(1, h)	12	9 : th
ing	(6, g)	12	10 : ing
s	(0, s)	12	11 : s
␣t	(8, t)	12	

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k	(0, k)	11	5: k
in	(3, n)	11	6: in
g	(0, g)	11	7: g
␣	(0, ␣)	11	8: ␣
th	(1, h)	12	9: th
ing	(6, g)	12	10: ing
s	(0, s)	12	11: s
␣t	(8, t)	12	12: ␣t

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th	(1, h)	12	9 : th
ing	(6, g)	12	10 : ing
s	(0, s)	12	11 : s
␣t	(8, t)	12	12 : ␣t
hr	(2, r)	12	

Message:

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ing	(6, g)	12	10 : ing
s	(0, s)	12	11 : s
␣t	(8, t)	12	12 : ␣t
hr	(2, r)	12	13 : hr

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g	(0, g)	11	7: g
␣	(0, ␣)	11	8: ␣
th	(1, h)	12	9: th
ing	(6, g)	12	10: ing
s	(0, s)	12	11: s
␣t	(8, t)	12	12: ␣t
hr	(2, r)	12	13: hr
o	(0, o)	12	

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k	(0, k)	11	5: k
in	(3, n)	11	6: in
g	(0, g)	11	7: g
␣	(0, ␣)	11	8: ␣
th	(1, h)	12	9: th
ing	(6, g)	12	10: ing
s	(0, s)	12	11: s
␣t	(8, t)	12	12: ␣t
hr	(2, r)	12	13: hr
o	(0, o)	12	14: o

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n	(0, n)	10	4 : n
k	(0, k)	11	5 : k
in	(3, n)	11	6 : in
g	(0, g)	11	7 : g
␣	(0, ␣)	11	8 : ␣
th	(1, h)	12	9 : th
ing	(6, g)	12	10 : ing
s	(0, s)	12	11 : s
␣t	(8, t)	12	12 : ␣t
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o	(0, o)	12	14 : o
u	(0, u)	12	

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n	(0, n)	10	4: n
k	(0, k)	11	5: k
in	(3, n)	11	6: in
g	(0, g)	11	7: g
␣	(0, ␣)	11	8: ␣
th	(1, h)	12	9: th
ing	(6, g)	12	10: ing
s	(0, s)	12	11: s
␣t	(8, t)	12	12: ␣t
hr	(2, r)	12	13: hr
o	(0, o)	12	14: o
u	(0, u)	12	15: u

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g	(0, g)	11	7: g
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␣t	(8, t)	12	12: ␣t
hr	(2, r)	12	13: hr
o	(0, o)	12	14: o
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gh	(7, h)	12	

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s	(0, s)	12	11: s
␣t	(8, t)	12	12: ␣t
hr	(2, r)	12	13: hr
o	(0, o)	12	14: o
u	(0, u)	12	15: u
gh	(7, h)	12	16: gh

Message:

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th	(1, h)	12	9: th
ing	(6, g)	12	10: ing
s	(0, s)	12	11: s
␣t	(8, t)	12	12: ␣t
hr	(2, r)	12	13: hr
o	(0, o)	12	14: o
u	(0, u)	12	15: u
gh	(7, h)	12	16: gh

Message:

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Result:

- Original message: 184 bits (23 bytes)
- Required 177 bits in total

Remember: Number of bits for dictionary index k

$$n_k = \left\lceil \log_2(1 + \text{dictionary size}) \right\rceil$$

Toy Example: LZ78 Decoding

input	phrase	dictionary
(0, t)		
(0, h)		
(0, i)		
(0, n)		
(0, k)		
(3, n)		
(0, g)		
(0, ␣)		
(1, h)		
(6, g)		
(0, s)		
(8, t)		
(2, r)		
(0, o)		
(0, u)		
(7, h)		

Decoded Message:

Toy Example: LZ78 Decoding

input	phrase	dictionary
(0, t)	t	
(0, h)		
(0, i)		
(0, n)		
(0, k)		
(3, n)		
(0, g)		
(0, ␣)		
(1, h)		
(6, g)		
(0, s)		
(8, t)		
(2, r)		
(0, o)		
(0, u)		
(7, h)		

Decoded Message:

t

Toy Example: LZ78 Decoding

input	phrase	dictionary
(0, t)	t	1 : t
(0, h)		
(0, i)		
(0, n)		
(0, k)		
(3, n)		
(0, g)		
(0, ␣)		
(1, h)		
(6, g)		
(0, s)		
(8, t)		
(2, r)		
(0, o)		
(0, u)		
(7, h)		

Decoded Message:

t

Toy Example: LZ78 Decoding

input	phrase	dictionary
(0, t)	t	1: t
(0, h)	h	
(0, i)		
(0, n)		
(0, k)		
(3, n)		
(0, g)		
(0, ␣)		
(1, h)		
(6, g)		
(0, s)		
(8, t)		
(2, r)		
(0, o)		
(0, u)		
(7, h)		

Decoded Message:

th

Toy Example: LZ78 Decoding

input	phrase	dictionary
(0, t)	t	1: t
(0, h)	h	2: h
(0, i)		
(0, n)		
(0, k)		
(3, n)		
(0, g)		
(0, ␣)		
(1, h)		
(6, g)		
(0, s)		
(8, t)		
(2, r)		
(0, o)		
(0, u)		
(7, h)		

Decoded Message:

th

Toy Example: LZ78 Decoding

input	phrase	dictionary
(0, t)	t	1: t
(0, h)	h	2: h
(0, i)	i	
(0, n)		
(0, k)		
(3, n)		
(0, g)		
(0, ␣)		
(1, h)		
(6, g)		
(0, s)		
(8, t)		
(2, r)		
(0, o)		
(0, u)		
(7, h)		

Decoded Message:

thi

Toy Example: LZ78 Decoding

input	phrase	dictionary
(0, t)	t	1 : t
(0, h)	h	2 : h
(0, i)	i	3 : i
(0, n)		
(0, k)		
(3, n)		
(0, g)		
(0, ␣)		
(1, h)		
(6, g)		
(0, s)		
(8, t)		
(2, r)		
(0, o)		
(0, u)		
(7, h)		

Decoded Message:

thi

Toy Example: LZ78 Decoding

input	phrase	dictionary
(0, t)	t	1 : t
(0, h)	h	2 : h
(0, i)	i	3 : i
(0, n)	n	
(0, k)		
(3, n)		
(0, g)		
(0, ␣)		
(1, h)		
(6, g)		
(0, s)		
(8, t)		
(2, r)		
(0, o)		
(0, u)		
(7, h)		

Decoded Message:

thin

Toy Example: LZ78 Decoding

input	phrase	dictionary
(0, t)	t	1 : t
(0, h)	h	2 : h
(0, i)	i	3 : i
(0, n)	n	4 : n
(0, k)		
(3, n)		
(0, g)		
(0, ␣)		
(1, h)		
(6, g)		
(0, s)		
(8, t)		
(2, r)		
(0, o)		
(0, u)		
(7, h)		

Decoded Message:

thin

Toy Example: LZ78 Decoding

input	phrase	dictionary
(0, t)	t	1 : t
(0, h)	h	2 : h
(0, i)	i	3 : i
(0, n)	n	4 : n
(0, k)	k	
(3, n)		
(0, g)		
(0, ␣)		
(1, h)		
(6, g)		
(0, s)		
(8, t)		
(2, r)		
(0, o)		
(0, u)		
(7, h)		

Decoded Message:

think

Toy Example: LZ78 Decoding

input	phrase	dictionary
(0, t)	t	1 : t
(0, h)	h	2 : h
(0, i)	i	3 : i
(0, n)	n	4 : n
(0, k)	k	5 : k
(3, n)		
(0, g)		
(0, ␣)		
(1, h)		
(6, g)		
(0, s)		
(8, t)		
(2, r)		
(0, o)		
(0, u)		
(7, h)		

Decoded Message:

think

Toy Example: LZ78 Decoding

input	phrase	dictionary
(0, t)	t	1 : t
(0, h)	h	2 : h
(0, i)	i	3 : i
(0, n)	n	4 : n
(0, k)	k	5 : k
(3, n)	in	
(0, g)		
(0, ␣)		
(1, h)		
(6, g)		
(0, s)		
(8, t)		
(2, r)		
(0, o)		
(0, u)		
(7, h)		

Decoded Message:

thinkin

Toy Example: LZ78 Decoding

input	phrase	dictionary
(0, t)	t	1: t
(0, h)	h	2: h
(0, i)	i	3: i
(0, n)	n	4: n
(0, k)	k	5: k
(3, n)	in	6: in
(0, g)		
(0, ␣)		
(1, h)		
(6, g)		
(0, s)		
(8, t)		
(2, r)		
(0, o)		
(0, u)		
(7, h)		

Decoded Message:

thinkin

Toy Example: LZ78 Decoding

input	phrase	dictionary
(0, t)	t	1 : t
(0, h)	h	2 : h
(0, i)	i	3 : i
(0, n)	n	4 : n
(0, k)	k	5 : k
(3, n)	in	6 : in
(0, g)	g	
(0, ␣)		
(1, h)		
(6, g)		
(0, s)		
(8, t)		
(2, r)		
(0, o)		
(0, u)		
(7, h)		

Decoded Message:

thinking

Toy Example: LZ78 Decoding

input	phrase	dictionary
(0, t)	t	1 : t
(0, h)	h	2 : h
(0, i)	i	3 : i
(0, n)	n	4 : n
(0, k)	k	5 : k
(3, n)	in	6 : in
(0, g)	g	7 : g
(0, ␣)		
(1, h)		
(6, g)		
(0, s)		
(8, t)		
(2, r)		
(0, o)		
(0, u)		
(7, h)		

Decoded Message:

thinking

Toy Example: LZ78 Decoding

input	phrase	dictionary
(0, t)	t	1 : t
(0, h)	h	2 : h
(0, i)	i	3 : i
(0, n)	n	4 : n
(0, k)	k	5 : k
(3, n)	in	6 : in
(0, g)	g	7 : g
(0, ␣)	␣	
(1, h)		
(6, g)		
(0, s)		
(8, t)		
(2, r)		
(0, o)		
(0, u)		
(7, h)		

Decoded Message:

thinking␣

Toy Example: LZ78 Decoding

input	phrase	dictionary
(0, t)	t	1 : t
(0, h)	h	2 : h
(0, i)	i	3 : i
(0, n)	n	4 : n
(0, k)	k	5 : k
(3, n)	in	6 : in
(0, g)	g	7 : g
(0, ␣)	␣	8 : ␣
(1, h)		
(6, g)		
(0, s)		
(8, t)		
(2, r)		
(0, o)		
(0, u)		
(7, h)		

Decoded Message:

thinking␣

Toy Example: LZ78 Decoding

input	phrase	dictionary
(0, t)	t	1: t
(0, h)	h	2: h
(0, i)	i	3: i
(0, n)	n	4: n
(0, k)	k	5: k
(3, n)	in	6: in
(0, g)	g	7: g
(0, ␣)	␣	8: ␣
(1, h)	th	
(6, g)		
(0, s)		
(8, t)		
(2, r)		
(0, o)		
(0, u)		
(7, h)		

Decoded Message:

thinking␣th

Toy Example: LZ78 Decoding

input	phrase	dictionary
(0, t)	t	1 : t
(0, h)	h	2 : h
(0, i)	i	3 : i
(0, n)	n	4 : n
(0, k)	k	5 : k
(3, n)	in	6 : in
(0, g)	g	7 : g
(0, ␣)	␣	8 : ␣
(1, h)	th	9 : th
(6, g)		
(0, s)		
(8, t)		
(2, r)		
(0, o)		
(0, u)		
(7, h)		

Decoded Message:

thinking␣th

Toy Example: LZ78 Decoding

input	phrase	dictionary
(0, t)	t	1 : t
(0, h)	h	2 : h
(0, i)	i	3 : i
(0, n)	n	4 : n
(0, k)	k	5 : k
(3, n)	in	6 : in
(0, g)	g	7 : g
(0, ␣)	␣	8 : ␣
(1, h)	th	9 : th
(6, g)	ing	
(0, s)		
(8, t)		
(2, r)		
(0, o)		
(0, u)		
(7, h)		

Decoded Message:

thinking␣thing

Toy Example: LZ78 Decoding

input	phrase	dictionary
(0, t)	t	1: t
(0, h)	h	2: h
(0, i)	i	3: i
(0, n)	n	4: n
(0, k)	k	5: k
(3, n)	in	6: in
(0, g)	g	7: g
(0, ␣)	␣	8: ␣
(1, h)	th	9: th
(6, g)	ing	10: ing
(0, s)		
(8, t)		
(2, r)		
(0, o)		
(0, u)		
(7, h)		

Decoded Message:

thinking␣thing

Toy Example: LZ78 Decoding

input	phrase	dictionary
(0, t)	t	1 : t
(0, h)	h	2 : h
(0, i)	i	3 : i
(0, n)	n	4 : n
(0, k)	k	5 : k
(3, n)	in	6 : in
(0, g)	g	7 : g
(0, ␣)	␣	8 : ␣
(1, h)	th	9 : th
(6, g)	ing	10 : ing
(0, s)	s	
(8, t)		
(2, r)		
(0, o)		
(0, u)		
(7, h)		

Decoded Message:

thinking␣things

Toy Example: LZ78 Decoding

input	phrase	dictionary
(0, t)	t	1: t
(0, h)	h	2: h
(0, i)	i	3: i
(0, n)	n	4: n
(0, k)	k	5: k
(3, n)	in	6: in
(0, g)	g	7: g
(0, ␣)	␣	8: ␣
(1, h)	th	9: th
(6, g)	ing	10: ing
(0, s)	s	11: s
(8, t)		
(2, r)		
(0, o)		
(0, u)		
(7, h)		

Decoded Message:

thinking␣things

Toy Example: LZ78 Decoding

input	phrase	dictionary
(0, t)	t	1 : t
(0, h)	h	2 : h
(0, i)	i	3 : i
(0, n)	n	4 : n
(0, k)	k	5 : k
(3, n)	in	6 : in
(0, g)	g	7 : g
(0, ␣)	␣	8 : ␣
(1, h)	th	9 : th
(6, g)	ing	10 : ing
(0, s)	s	11 : s
(8, t)	␣t	
(2, r)		
(0, o)		
(0, u)		
(7, h)		

Decoded Message:

thinking␣things␣t

Toy Example: LZ78 Decoding

input	phrase	dictionary
(0, t)	t	1 : t
(0, h)	h	2 : h
(0, i)	i	3 : i
(0, n)	n	4 : n
(0, k)	k	5 : k
(3, n)	in	6 : in
(0, g)	g	7 : g
(0, ␣)	␣	8 : ␣
(1, h)	th	9 : th
(6, g)	ing	10 : ing
(0, s)	s	11 : s
(8, t)	␣t	12 : ␣t
(2, r)		
(0, o)		
(0, u)		
(7, h)		

Decoded Message:

thinking␣things␣t

Toy Example: LZ78 Decoding

input	phrase	dictionary
(0, t)	t	1 : t
(0, h)	h	2 : h
(0, i)	i	3 : i
(0, n)	n	4 : n
(0, k)	k	5 : k
(3, n)	in	6 : in
(0, g)	g	7 : g
(0, ␣)	␣	8 : ␣
(1, h)	th	9 : th
(6, g)	ing	10 : ing
(0, s)	s	11 : s
(8, t)	␣t	12 : ␣t
(2, r)	hr	
(0, o)		
(0, u)		
(7, h)		

Decoded Message:

thinking␣things␣thr

Toy Example: LZ78 Decoding

input	phrase	dictionary
(0, t)	t	1: t
(0, h)	h	2: h
(0, i)	i	3: i
(0, n)	n	4: n
(0, k)	k	5: k
(3, n)	in	6: in
(0, g)	g	7: g
(0, ␣)	␣	8: ␣
(1, h)	th	9: th
(6, g)	ing	10: ing
(0, s)	s	11: s
(8, t)	␣t	12: ␣t
(2, r)	hr	13: hr
(0, o)		
(0, u)		
(7, h)		

Decoded Message:

thinking␣things␣thr

Toy Example: LZ78 Decoding

input	phrase	dictionary
(0, t)	t	1: t
(0, h)	h	2: h
(0, i)	i	3: i
(0, n)	n	4: n
(0, k)	k	5: k
(3, n)	in	6: in
(0, g)	g	7: g
(0, ␣)	␣	8: ␣
(1, h)	th	9: th
(6, g)	ing	10: ing
(0, s)	s	11: s
(8, t)	␣t	12: ␣t
(2, r)	hr	13: hr
(0, o)	o	
(0, u)		
(7, h)		

Decoded Message:

thinking␣things␣thro

Toy Example: LZ78 Decoding

input	phrase	dictionary
(0, t)	t	1: t
(0, h)	h	2: h
(0, i)	i	3: i
(0, n)	n	4: n
(0, k)	k	5: k
(3, n)	in	6: in
(0, g)	g	7: g
(0, ␣)	␣	8: ␣
(1, h)	th	9: th
(6, g)	ing	10: ing
(0, s)	s	11: s
(8, t)	␣t	12: ␣t
(2, r)	hr	13: hr
(0, o)	o	14: o
(0, u)		
(7, h)		

Decoded Message:

thinking␣things␣thro

Toy Example: LZ78 Decoding

input	phrase	dictionary
(0, t)	t	1 : t
(0, h)	h	2 : h
(0, i)	i	3 : i
(0, n)	n	4 : n
(0, k)	k	5 : k
(3, n)	in	6 : in
(0, g)	g	7 : g
(0, ␣)	␣	8 : ␣
(1, h)	th	9 : th
(6, g)	ing	10 : ing
(0, s)	s	11 : s
(8, t)	␣t	12 : ␣t
(2, r)	hr	13 : hr
(0, o)	o	14 : o
(0, u)	u	
(7, h)		

Decoded Message:

thinking␣things␣throu

Toy Example: LZ78 Decoding

input	phrase	dictionary
(0, t)	t	1 : t
(0, h)	h	2 : h
(0, i)	i	3 : i
(0, n)	n	4 : n
(0, k)	k	5 : k
(3, n)	in	6 : in
(0, g)	g	7 : g
(0, ␣)	␣	8 : ␣
(1, h)	th	9 : th
(6, g)	ing	10 : ing
(0, s)	s	11 : s
(8, t)	␣t	12 : ␣t
(2, r)	hr	13 : hr
(0, o)	o	14 : o
(0, u)	u	15 : u
(7, h)		

Decoded Message:

thinking␣things␣throu

Toy Example: LZ78 Decoding

input	phrase	dictionary
(0, t)	t	1: t
(0, h)	h	2: h
(0, i)	i	3: i
(0, n)	n	4: n
(0, k)	k	5: k
(3, n)	in	6: in
(0, g)	g	7: g
(0, ␣)	␣	8: ␣
(1, h)	th	9: th
(6, g)	ing	10: ing
(0, s)	s	11: s
(8, t)	␣t	12: ␣t
(2, r)	hr	13: hr
(0, o)	o	14: o
(0, u)	u	15: u
(7, h)	gh	

Decoded Message:

thinking␣things␣through

Toy Example: LZ78 Decoding

input	phrase	dictionary
(0, t)	t	1 : t
(0, h)	h	2 : h
(0, i)	i	3 : i
(0, n)	n	4 : n
(0, k)	k	5 : k
(3, n)	in	6 : in
(0, g)	g	7 : g
(0, ␣)	␣	8 : ␣
(1, h)	th	9 : th
(6, g)	ing	10 : ing
(0, s)	s	11 : s
(8, t)	␣t	12 : ␣t
(2, r)	hr	13 : hr
(0, o)	o	14 : o
(0, u)	u	15 : u
(7, h)	gh	16 : gh

Decoded Message:

thinking␣things␣through

LZ78 Variant: The Lempel-Ziv-Welch Algorithm (LZW)

Main Difference to LZ78

- Dictionary is initialized with all strings of length one (i.e., all byte codes)
- Next symbol is not included in code

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The LZW Algorithm

- Send code for dictionary entry that matches start of remaining sequence
- After sending a code, a new dictionary entry is added that consists of
  - the phrases that was just coded followed by
  - the next symbol in the message

LZ78 Variant: The Lempel-Ziv-Welch Algorithm (LZW)

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The LZW Algorithm

- Send code for dictionary entry that matches start of remaining sequence
- After sending a code, a new dictionary entry is added that consists of
    - the phrases that was just coded followed by
    - the next symbol in the message

Applications using the LZW Algorithm

- Unix file compression tool **compress**
- Image coding format **GIF**
- Optional compression mode in **PDF** and **TIFF**

Toy Example: LZW Encoding

phrase	next	output	dictionary

Message:

`thinking␣things␣through`

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Encoding

phrase	next	output	dictionary
t	h	<116>	

Message:

thinking␣things␣through

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Encoding

phrase	next	output	dictionary
t	h	<116>	256: th

Message:

thinking␣things␣through

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Encoding

phrase	next	output	dictionary
t	h	<116>	256: th
h	i	<104>	

Message:

thinking␣things␣through

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Encoding

phrase	next	output	dictionary
t	h	<116>	256: th
h	i	<104>	257: hi

Message:

thinking␣things␣through

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Encoding

phrase	next	output	dictionary
t	h	<116>	256: th
h	i	<104>	257: hi
i	n	<105>	

Message:

thinking␣things␣through

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Encoding

phrase	next	output	dictionary
t	h	<116>	256: th
h	i	<104>	257: hi
i	n	<105>	258: in

Message:

thinking␣things␣through

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Encoding

phrase	next	output	dictionary
t	h	<116>	256: th
h	i	<104>	257: hi
i	n	<105>	258: in
n	k	<110>	

Message:

thinking␣things␣through

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Encoding

phrase	next	output	dictionary
t	h	<116>	256: th
h	i	<104>	257: hi
i	n	<105>	258: in
n	k	<110>	259: nk

Message:

thinking␣things␣through

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Encoding

phrase	next	output	dictionary
t	h	<116>	256: th
h	i	<104>	257: hi
i	n	<105>	258: in
n	k	<110>	259: nk
k	i	<107>	

Message:

thinking␣things␣through

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Encoding

phrase	next	output	dictionary
t	h	<116>	256: th
h	i	<104>	257: hi
i	n	<105>	258: in
n	k	<110>	259: nk
k	i	<107>	260: ki

Message:

thinking␣things␣through

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Encoding

phrase	next	output	dictionary
t	h	<116>	256: th
h	i	<104>	257: hi
i	n	<105>	258: in
n	k	<110>	259: nk
k	i	<107>	260: ki
in	g	<258>	

Message:

thinking␣things␣through

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Encoding

phrase	next	output	dictionary
t	h	<116>	256: th
h	i	<104>	257: hi
i	n	<105>	258: in
n	k	<110>	259: nk
k	i	<107>	260: ki
in	g	<258>	261: ing

Message:

thinking␣things␣through

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Encoding

phrase	next	output	dictionary
t	h	<116>	256: th
h	i	<104>	257: hi
i	n	<105>	258: in
n	k	<110>	259: nk
k	i	<107>	260: ki
in	g	<258>	261: ing
g	␣	<103>	

Message:

thinking␣things␣through

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Encoding

phrase	next	output	dictionary
t	h	<116>	256: th
h	i	<104>	257: hi
i	n	<105>	258: in
n	k	<110>	259: nk
k	i	<107>	260: ki
in	g	<258>	261: ing
g	␣	<103>	262: g␣

Message:

thinking␣things␣through

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Encoding

phrase	next	output	dictionary
t	h	<116>	256: th
h	i	<104>	257: hi
i	n	<105>	258: in
n	k	<110>	259: nk
k	i	<107>	260: ki
in	g	<258>	261: ing
g	␣	<103>	262: g␣
␣	t	<32>	

Message:

thinking␣things␣through

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Encoding

phrase	next	output	dictionary
t	h	<116>	256: th
h	i	<104>	257: hi
i	n	<105>	258: in
n	k	<110>	259: nk
k	i	<107>	260: ki
in	g	<258>	261: ing
g	␣	<103>	262: g␣
␣	t	<32>	263: ␣t

Message:

thinking␣things␣through

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Encoding

phrase	next	output	dictionary
t	h	<116>	256: th
h	i	<104>	257: hi
i	n	<105>	258: in
n	k	<110>	259: nk
k	i	<107>	260: ki
in	g	<258>	261: ing
g	␣	<103>	262: g␣
␣	t	<32>	263: ␣t
th	i	<256>	

Message:

thinking␣things␣through

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Encoding

phrase	next	output	dictionary
t	h	<116>	256: th
h	i	<104>	257: hi
i	n	<105>	258: in
n	k	<110>	259: nk
k	i	<107>	260: ki
in	g	<258>	261: ing
g	␣	<103>	262: g␣
␣	t	<32>	263: ␣t
th	i	<256>	264: thi

Message:

thinking␣things␣through

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Encoding

phrase	next	output	dictionary
t	h	<116>	256: th
h	i	<104>	257: hi
i	n	<105>	258: in
n	k	<110>	259: nk
k	i	<107>	260: ki
in	g	<258>	261: ing
g	␣	<103>	262: g␣
␣	t	<32>	263: ␣t
th	i	<256>	264: thi
ing	s	<261>	

Message:

thinking␣things␣through

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Encoding

phrase	next	output	dictionary
t	h	<116>	256: th
h	i	<104>	257: hi
i	n	<105>	258: in
n	k	<110>	259: nk
k	i	<107>	260: ki
in	g	<258>	261: ing
g	␣	<103>	262: g␣
␣	t	<32>	263: ␣t
th	i	<256>	264: thi
ing	s	<261>	265: ings

Message:

thinking␣things␣through

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Encoding

phrase	next	output	dictionary
t	h	<116>	256: th
h	i	<104>	257: hi
i	n	<105>	258: in
n	k	<110>	259: nk
k	i	<107>	260: ki
in	g	<258>	261: ing
g	␣	<103>	262: g␣
␣	t	<32>	263: ␣t
th	i	<256>	264: thi
ing	s	<261>	265: ings
s	␣	<115>	

Message:

thinking␣things␣through

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Encoding

phrase	next	output	dictionary
t	h	<116>	256: th
h	i	<104>	257: hi
i	n	<105>	258: in
n	k	<110>	259: nk
k	i	<107>	260: ki
in	g	<258>	261: ing
g	␣	<103>	262: g␣
␣	t	<32>	263: ␣t
th	i	<256>	264: thi
ing	s	<261>	265: ings
s	␣	<115>	266: s␣

Message:

thinking␣things␣through

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Encoding

phrase	next	output	dictionary
t	h	<116>	256: th
h	i	<104>	257: hi
i	n	<105>	258: in
n	k	<110>	259: nk
k	i	<107>	260: ki
in	g	<258>	261: ing
g	␣	<103>	262: g␣
␣	t	<32>	263: ␣t
th	i	<256>	264: thi
ing	s	<261>	265: ings
s	␣	<115>	266: s␣
␣t	h	<263>	

Message:

thinking␣things␣through

Pre-initialized dictionary:

- All byte codes: <0> … <255>

Toy Example: LZW Encoding

phrase	next	output	dictionary
t	h	<116>	256: th
h	i	<104>	257: hi
i	n	<105>	258: in
n	k	<110>	259: nk
k	i	<107>	260: ki
in	g	<258>	261: ing
g	␣	<103>	262: g␣
␣	t	<32>	263: ␣t
th	i	<256>	264: thi
ing	s	<261>	265: ings
s	␣	<115>	266: s␣
␣t	h	<263>	267: ␣th

Message:

thinking␣things␣through

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Encoding

phrase	next	output	dictionary
t	h	<116>	256: th
h	i	<104>	257: hi
i	n	<105>	258: in
n	k	<110>	259: nk
k	i	<107>	260: ki
in	g	<258>	261: ing
g	␣	<103>	262: g␣
␣	t	<32>	263: ␣t
th	i	<256>	264: thi
ing	s	<261>	265: ings
s	␣	<115>	266: s␣
␣t	h	<263>	267: ␣th
h	r	<104>	

Message:

thinking␣things␣through

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Encoding

phrase	next	output	dictionary
t	h	<116>	256: th
h	i	<104>	257: hi
i	n	<105>	258: in
n	k	<110>	259: nk
k	i	<107>	260: ki
in	g	<258>	261: ing
g	␣	<103>	262: g␣
␣	t	<32>	263: ␣t
th	i	<256>	264: thi
ing	s	<261>	265: ings
s	␣	<115>	266: s␣
␣t	h	<263>	267: ␣th
h	r	<104>	268: hr

Message:

thinking␣things␣through

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Encoding

phrase	next	output	dictionary
t	h	<116>	256: th
h	i	<104>	257: hi
i	n	<105>	258: in
n	k	<110>	259: nk
k	i	<107>	260: ki
in	g	<258>	261: ing
g	␣	<103>	262: g␣
␣	t	<32>	263: ␣t
th	i	<256>	264: thi
ing	s	<261>	265: ings
s	␣	<115>	266: s␣
␣t	h	<263>	267: ␣th
h	r	<104>	268: hr
r	o	<114>	

Message:

thinking␣things␣through

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Encoding

phrase	next	output	dictionary
t	h	<116>	256: th
h	i	<104>	257: hi
i	n	<105>	258: in
n	k	<110>	259: nk
k	i	<107>	260: ki
in	g	<258>	261: ing
g	␣	<103>	262: g␣
␣	t	<32>	263: ␣t
th	i	<256>	264: thi
ing	s	<261>	265: ings
s	␣	<115>	266: s␣
␣t	h	<263>	267: ␣th
h	r	<104>	268: hr
r	o	<114>	269: ro

Message:

thinking␣things␣through

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Encoding

phrase	next	output	dictionary
t	h	<116>	256: th
h	i	<104>	257: hi
i	n	<105>	258: in
n	k	<110>	259: nk
k	i	<107>	260: ki
in	g	<258>	261: ing
g	␣	<103>	262: g␣
␣	t	<32>	263: ␣t
th	i	<256>	264: thi
ing	s	<261>	265: ings
s	␣	<115>	266: s␣
␣t	h	<263>	267: ␣th
h	r	<104>	268: hr
r	o	<114>	269: ro
o	u	<111>	

Message:

thinking␣things␣through

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Encoding

phrase	next	output	dictionary
t	h	<116>	256: th
h	i	<104>	257: hi
i	n	<105>	258: in
n	k	<110>	259: nk
k	i	<107>	260: ki
in	g	<258>	261: ing
g	␣	<103>	262: g␣
␣	t	<32>	263: ␣t
th	i	<256>	264: thi
ing	s	<261>	265: ings
s	␣	<115>	266: s␣
␣t	h	<263>	267: ␣th
h	r	<104>	268: hr
r	o	<114>	269: ro
o	u	<111>	270: ou

Message:

thinking␣things␣through

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Encoding

phrase	next	output	dictionary
t	h	<116>	256: th
h	i	<104>	257: hi
i	n	<105>	258: in
n	k	<110>	259: nk
k	i	<107>	260: ki
in	g	<258>	261: ing
g	␣	<103>	262: g␣
␣	t	<32>	263: ␣t
th	i	<256>	264: thi
ing	s	<261>	265: ings
s	␣	<115>	266: s␣
␣t	h	<263>	267: ␣th
h	r	<104>	268: hr
r	o	<114>	269: ro
o	u	<111>	270: ou
u	g	<117>	

Message:

thinking␣things␣through

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Encoding

phrase	next	output	dictionary
t	h	<116>	256: th
h	i	<104>	257: hi
i	n	<105>	258: in
n	k	<110>	259: nk
k	i	<107>	260: ki
in	g	<258>	261: ing
g	␣	<103>	262: g␣
␣	t	<32>	263: ␣t
th	i	<256>	264: thi
ing	s	<261>	265: ings
s	␣	<115>	266: s␣
␣t	h	<263>	267: ␣th
h	r	<104>	268: hr
r	o	<114>	269: ro
o	u	<111>	270: ou
u	g	<117>	271: ug

Message:

thinking␣things␣through

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Encoding

phrase	next	output	dictionary
t	h	<116>	256: th
h	i	<104>	257: hi
i	n	<105>	258: in
n	k	<110>	259: nk
k	i	<107>	260: ki
in	g	<258>	261: ing
g	␣	<103>	262: g␣
␣	t	<32>	263: ␣t
th	i	<256>	264: thi
ing	s	<261>	265: ings
s	␣	<115>	266: s␣
␣t	h	<263>	267: ␣th
h	r	<104>	268: hr
r	o	<114>	269: ro
o	u	<111>	270: ou
u	g	<117>	271: ug
g	h	<103>	

Message:

thinking␣things␣through

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Encoding

phrase	next	output	dictionary
t	h	<116>	256: th
h	i	<104>	257: hi
i	n	<105>	258: in
n	k	<110>	259: nk
k	i	<107>	260: ki
in	g	<258>	261: ing
g	␣	<103>	262: g␣
␣	t	<32>	263: ␣t
th	i	<256>	264: thi
ing	s	<261>	265: ings
s	␣	<115>	266: s␣
␣t	h	<263>	267: ␣th
h	r	<104>	268: hr
r	o	<114>	269: ro
o	u	<111>	270: ou
u	g	<117>	271: ug
g	h	<103>	272: gh

Message:

thinking␣things␣through

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Encoding

phrase	next	output	dictionary
t	h	<116>	256: th
h	i	<104>	257: hi
i	n	<105>	258: in
n	k	<110>	259: nk
k	i	<107>	260: ki
in	g	<258>	261: ing
g	␣	<103>	262: g␣
␣	t	<32>	263: ␣t
th	i	<256>	264: thi
ing	s	<261>	265: ings
s	␣	<115>	266: s␣
␣t	h	<263>	267: ␣th
h	r	<104>	268: hr
r	o	<114>	269: ro
o	u	<111>	270: ou
u	g	<117>	271: ug
g	h	<103>	272: gh
h		<104>	

Message:

thinking␣things␣through

Pre-initialized dictionary:

- All byte codes: <0> … <255>

Toy Example: LZW Encoding

phrase	next	output	dictionary
t	h	<116>	256: th
h	i	<104>	257: hi
i	n	<105>	258: in
n	k	<110>	259: nk
k	i	<107>	260: ki
in	g	<258>	261: ing
g	␣	<103>	262: g␣
␣	t	<32>	263: ␣t
th	i	<256>	264: thi
ing	s	<261>	265: ings
s	␣	<115>	266: s␣
␣t	h	<263>	267: ␣th
h	r	<104>	268: hr
r	o	<114>	269: ro
o	u	<111>	270: ou
u	g	<117>	271: ug
g	h	<103>	272: gh
h		<104>	273: h

Message:

thinking␣things␣through

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Encoding

phrase	next	output	dictionary
t	h	<116>	256: th
h	i	<104>	257: hi
i	n	<105>	258: in
n	k	<110>	259: nk
k	i	<107>	260: ki
in	g	<258>	261: ing
g	␣	<103>	262: g␣
␣	t	<32>	263: ␣t
th	i	<256>	264: thi
ing	s	<261>	265: ings
s	␣	<115>	266: s␣
␣t	h	<263>	267: ␣th
h	r	<104>	268: hr
r	o	<114>	269: ro
o	u	<111>	270: ou
u	g	<117>	271: ug
g	h	<103>	272: gh
h		<104>	273: h

Message:

thinking␣things␣through

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Result:

- Original message: 184 bits (23 bytes)
- Required 162 bits (18×9 bits)

Toy Example: LZW Decoding

input	output	dictionary	conjecture

Message:

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Decoding

input	output	dictionary	conjecture
<116>	t		

Message:

t

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Decoding

input	output	dictionary	conjecture
<116>	t		256: t?

Message:

t

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Decoding

input	output	dictionary	conjecture
<116>	t		256: t?
<104>	h		

Message:

th

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Decoding

input	output	dictionary	conjecture
<116>	t		256: t?
<104>	h	256: th	257: h?

Message:

th

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Decoding

input	output	dictionary	conjecture
<116>	t		256: t?
<104>	h	256: th	257: h?
<105>	i		

Message:

thi

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Decoding

input	output	dictionary	conjecture
<116>	t		256: t?
<104>	h	256: th	257: h?
<105>	i	257: hi	258: i?

Message:

thi

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Decoding

input	output	dictionary	conjecture
<116>	t		256: t?
<104>	h	256: th	257: h?
<105>	i	257: hi	258: i?
<110>	n		

Message:

thin

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Decoding

input	output	dictionary	conjecture
<116>	t		256: t?
<104>	h	256: th	257: h?
<105>	i	257: hi	258: i?
<110>	n	258: in	259: n?

Message:

thin

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Decoding

input	output	dictionary	conjecture
<116>	t		256: t?
<104>	h	256: th	257: h?
<105>	i	257: hi	258: i?
<110>	n	258: in	259: n?
<107>	k		

Message:

think

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Decoding

input	output	dictionary	conjecture
<116>	t		256: t?
<104>	h	256: th	257: h?
<105>	i	257: hi	258: i?
<110>	n	258: in	259: n?
<107>	k	259: nk	260: k?

Message:

think

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Decoding

input	output	dictionary	conjecture
<116>	t		256: t?
<104>	h	256: th	257: h?
<105>	i	257: hi	258: i?
<110>	n	258: in	259: n?
<107>	k	259: nk	260: k?
<258>	in		

Message:

thinkin

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Decoding

input	output	dictionary	conjecture
<116>	t		256: t?
<104>	h	256: th	257: h?
<105>	i	257: hi	258: i?
<110>	n	258: in	259: n?
<107>	k	259: nk	260: k?
<258>	in	260: ki	261: in?

Message:

thinkin

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Decoding

input	output	dictionary	conjecture
<116>	t		256: t?
<104>	h	256: th	257: h?
<105>	i	257: hi	258: i?
<110>	n	258: in	259: n?
<107>	k	259: nk	260: k?
<258>	in	260: ki	261: in?
<103>	g		

Message:

thinking

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Decoding

input	output	dictionary	conjecture
<116>	t		256: t?
<104>	h	256: th	257: h?
<105>	i	257: hi	258: i?
<110>	n	258: in	259: n?
<107>	k	259: nk	260: k?
<258>	in	260: ki	261: in?
<103>	g	261: ing	262: g?

Message:

thinking

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Decoding

input	output	dictionary	conjecture
<116>	t		256: t?
<104>	h	256: th	257: h?
<105>	i	257: hi	258: i?
<110>	n	258: in	259: n?
<107>	k	259: nk	260: k?
<258>	in	260: ki	261: in?
<103>	g	261: ing	262: g?
<32>	␣		

Message:

thinking␣

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Decoding

input	output	dictionary	conjecture
<116>	t		256: t?
<104>	h	256: th	257: h?
<105>	i	257: hi	258: i?
<110>	n	258: in	259: n?
<107>	k	259: nk	260: k?
<258>	in	260: ki	261: in?
<103>	g	261: ing	262: g?
<32>	␣	262: g␣	263: ␣?

Message:

thinking␣

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Decoding

input	output	dictionary	conjecture
<116>	t		256: t?
<104>	h	256: th	257: h?
<105>	i	257: hi	258: i?
<110>	n	258: in	259: n?
<107>	k	259: nk	260: k?
<258>	in	260: ki	261: in?
<103>	g	261: ing	262: g?
<32>	␣	262: g␣	263: ␣?
<256>	th		

Message:

thinking␣th

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Decoding

input	output	dictionary	conjecture
<116>	t		256: t?
<104>	h	256: th	257: h?
<105>	i	257: hi	258: i?
<110>	n	258: in	259: n?
<107>	k	259: nk	260: k?
<258>	in	260: ki	261: in?
<103>	g	261: ing	262: g?
<32>	␣	262: g␣	263: ␣?
<256>	th	263: ␣t	264: th?

Message:

thinking␣th

Pre-initialized dictionary:

- All byte codes: <0> ... <255>

Toy Example: LZW Decoding

input	output	dictionary	conjecture
<116>	t		256: t?
<104>	h	256: th	257: h?
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<107>	k	259: nk	260: k?
<258>	in	260: ki	261: in?
<103>	g	261: ing	262: g?
<32>	␣	262: g␣	263: ␣?
<256>	th	263: ␣t	264: th?
<261>	ing		

Message:

thinking␣thing

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<256>	th	263: ␣t	264: th?
<261>	ing	264: thi	265: ing?

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<256>	th	263: ␣t	264: th?
<261>	ing	264: thi	265: ing?
<115>	s		

Message:

thinking␣things

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<107>	k	259: nk	260: k?
<258>	in	260: ki	261: in?
<103>	g	261: ing	262: g?
<32>	␣	262: g␣	263: ␣?
<256>	th	263: ␣t	264: th?
<261>	ing	264: thi	265: ing?
<115>	s	265: ings	266: s?

Message:

thinking␣things

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<261>	ing	264: thi	265: ing?
<115>	s	265: ings	266: s?
<263>	␣t		

Message:

thinking␣things␣t

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<256>	th	263: ␣t	264: th?
<261>	ing	264: thi	265: ing?
<115>	s	265: ings	266: s?
<263>	␣t	266: s␣	267: ␣t?

Message:

thinking␣things␣t

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<256>	th	263: ␣t	264: th?
<261>	ing	264: thi	265: ing?
<115>	s	265: ings	266: s?
<263>	␣t	266: s␣	267: ␣t?
<104>	h		

Message:

thinking␣things␣th

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<256>	th	263: ␣t	264: th?
<261>	ing	264: thi	265: ing?
<115>	s	265: ings	266: s?
<263>	␣t	266: s␣	267: ␣t?
<104>	h	267: ␣th	268: h?

Message:

thinking␣things␣th

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<104>	h	267: ␣th	268: h?
<114>	r		

Message:

thinking␣things␣thr

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<104>	h	267: ␣th	268: h?
<114>	r	268: hr	269: r?

Message:

thinking␣things␣thr

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<111>	o		

Message:

thinking␣things␣thro

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<104>	h	267: ␣th	268: h?
<114>	r	268: hr	269: r?
<111>	o	269: ro	270: o?

Message:

thinking␣things␣thro

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<104>	h	267: ␣th	268: h?
<114>	r	268: hr	269: r?
<111>	o	269: ro	270: o?
<117>	u		

Message:

thinking␣things␣throu

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<114>	r	268: hr	269: r?
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<117>	u	270: ou	271: u?

Message:

thinking␣things␣throu

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<263>	␣t	266: s␣	267: ␣t?
<104>	h	267: ␣th	268: h?
<114>	r	268: hr	269: r?
<111>	o	269: ro	270: o?
<117>	u	270: ou	271: u?
<103>	g		

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thinking␣things␣throug

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<104>	h	267: ␣th	268: h?
<114>	r	268: hr	269: r?
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Message:

thinking␣things␣throug

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Message:

thinking␣things␣through

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<117>	u	270: ou	271: u?
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Message:

thinking␣things␣through

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LZW: The K-Omega-K Problem

Property of LZW Algorithm

- Decoder is one step behind encoder in constructing dictionary
- Encoder might send code for not yet completed dictionary entry

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Example: Coding of sequence "...cXYZcXYZca..."

encoder

phrase	next	output	dictionary
			<300>: cXYZ

decoder

input	output	dictionary	conjecture
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			<300>: cXYZ
cXYZ	c	<300>	

decoder

input	output	dictionary	conjecture
		<300>: cXYZ	

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Example: Coding of sequence "...cXYZcXYZca..."

encoder

phrase	next	output	dictionary
			<300>: cXYZ
cXYZ	c	<300>	<400>: cXYZc

decoder

input	output	dictionary	conjecture
		<300>: cXYZ	

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encoder

phrase	next	output	dictionary
			<300>: cXYZ
cXYZ	c	<300>	<400>: cXYZc
cXYZc	a	<400>	

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input	output	dictionary	conjecture
		<300>: cXYZ	

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input	output	dictionary	conjecture
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cXYZc	a	<400>	<401>: cXYZca

decoder

input	output	dictionary	conjecture
		<300>: cXYZ	
<300>	cXYZ		<400>: cXYZ?

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phrase	next	output	dictionary
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cXYZc	a	<400>	<401>: cXYZca

decoder

input	output	dictionary	conjecture
		<300>: cXYZ	
<300>	cXYZ		<400>: cXYZ?
<400>	cXYZ?		

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input	output	dictionary	conjecture
		<300>: cXYZ	
<300>	cXYZ		<400>: cXYZ?
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How can the decoder correctly decode in such a case ?

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cXYZc	a	<400>	<401>: cXYZca

decoder

input	output	dictionary	conjecture
		<300>: cXYZ	
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cXYZc	a	<400>	<401>: cXYZca

decoder

input	output	dictionary	conjecture
		<300>: cXYZ	
<300>	cXYZ		<400>: cXYZ?
<400>	cXYZ?		

How can the decoder correctly decode in such a case ?

- Incomplete dictionary entry is last added entry
- This entry is used only if the first symbol of new sequence is the last symbol of incomplete entry

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phrase	next	output	dictionary
			<300>: cXYZ
cXYZ	c	<300>	<400>: cXYZc
cXYZc	a	<400>	<401>: cXYZca

decoder

input	output	dictionary	conjecture
		<300>: cXYZ	
<300>	cXYZ		<400>: cXYZ?
<400>	cXYZ?	(cXYZ? must be cXYZc)	

How can the decoder correctly decode in such a case ?

- Incomplete dictionary entry is last added entry
- This entry is used only if the first symbol of new sequence is the last symbol of incomplete entry

➡ Last symbol must be equal to first symbol ! (in our example: "cXYZ?" = "cXYZc")

The Burrows-Wheeler Transform (BWT)

1. Create all rotations of the original message

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Example: Message "BANANAMAN"

B A N A N A M A N

$\xrightarrow{\text{rotations}}$

The Burrows-Wheeler Transform (BWT)

1. Create all rotations of the original message

Example: Message "BANANAMAN"

rotations $\longrightarrow$

```
B A N A N A M A N
A N A N A M A N B
```

The Burrows-Wheeler Transform (BWT)

1. Create all rotations of the original message

Example: Message "BANANAMAN"

rotations →

```
B A N A N A M A N
A N A N A M A N B
N A N A M A N B A
```

The Burrows-Wheeler Transform (BWT)

1. Create all rotations of the original message

Example: Message "BANANAMAN"

rotations →

```
B A N A N A M A N
A N A N A M A N B
N A N A M A N B A
A N A M A N B A N
```

The Burrows-Wheeler Transform (BWT)

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Example: Message "BANANAMAN"

rotations →

```
B A N A N A M A N
A N A N A M A N B
N A N A M A N B A
A N A M A N B A N
N A M A N B A N A
```

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Example: Message "BANANAMAN"

rotations →

```
B A N A N A M A N
A N A N A M A N B
N A N A M A N B A
A N A M A N B A N
N A M A N B A N A
A M A N B A N A N
```

The Burrows-Wheeler Transform (BWT)

1. Create all rotations of the original message

Example: Message "BANANAMAN"

rotations →

```
B A N A N A M A N
A N A N A M A N B
N A N A M A N B A
A N A M A N B A N
N A M A N B A N A
A M A N B A N A N
M A N B A N A N A
```

The Burrows-Wheeler Transform (BWT)

1. Create all rotations of the original message

Example: Message "BANANAMAN"

rotations →

```
B A N A N A M A N
A N A N A M A N B
N A N A M A N B A
A N A M A N B A N
N A M A N B A N A
A M A N B A N A N
M A N B A N A N A
A N B A N A N A M
```

The Burrows-Wheeler Transform (BWT)

1. Create all rotations of the original message

Example: Message "BANANAMAN"

rotations →

```
B A N A N A M A N
A N A N A M A N B
N A N A M A N B A
A N A M A N B A N
N A M A N B A N A
A M A N B A N A N
M A N B A N A N A
A N B A N A N A M
N B A N A N A M A
```

The Burrows-Wheeler Transform (BWT)

1. Create all rotations of the original message
2. Sort all rotations in lexicographical order

Example: Message "BANANAMAN"

rotations →

```
BANANAMAN
ANANAMANB
NANAMANBA
ANAMANBAN
NAMANBANA
AMANBANAN
MANBANANA
ANBANANAM
NBANANAMA
```

The Burrows-Wheeler Transform (BWT)

1. Create all rotations of the original message
2. Sort all rotations in lexicographical order

Example: Message "BANANAMAN"

rotations →	sorting →
BANANAMAN	AMANBANAN
ANANAMANB	ANAMANBAN
NANAMANBA	ANANAMANB
ANAMANBAN	ANBANANAM
NAMANBANA	BANANAMAN
AMANBANAN	MANBANANA
MANBANANA	NAMANBANA
ANBANANAM	NANAMANBA
NBANANAMA	NBANANAMA

The Burrows-Wheeler Transform (BWT)

1. Create all rotations of the original message
2. Sort all rotations in lexicographical order
3. Output: Last column of the sorted block + index of original message (in sorted block)

Example: Message "BANANAMAN"

rotations →		sorting →	
	B A N A N A M A N		A M A N B A N A N
	A N A N A M A N B		A N A M A N B A N
	N A N A M A N B A		A N A N A M A N B
	A N A M A N B A N		A N B A N A N A M
	N A M A N B A N A		B A N A N A M A N
	A M A N B A N A N		M A N B A N A N A
	M A N B A N A N A		N A M A N B A N A
	A N B A N A N A M		N A N A M A N B A
	N B A N A N A M A		N B A N A N A M A

The Burrows-Wheeler Transform (BWT)

1. Create all rotations of the original message
2. Sort all rotations in lexicographical order
3. Output: Last column of the sorted block + index of original message (in sorted block)

Example: Message "BANANAMAN"

rotations →	BANANAMAN	sorting →	AMANBANAN	last column →	NNBMNAAAA
	ANANAMANB		ANAMANBAN		
	NANAMANBA		ANANAMANB		
	ANAMANBAN		ANBANANAM		
	NAMANBANA		BANANAMAN		
	AMANBANAN		MANBANANA		
	MANBANANA		NAMANBANA		
	ANBANANAM		NANAMANBA		
	NBANANAMA		NBANANAMA		

The Burrows-Wheeler Transform (BWT)

1. Create all rotations of the original message
2. Sort all rotations in lexicographical order
3. Output: Last column of the sorted block + index of original message (in sorted block)

Example: Message "BANANAMAN"

rotations →	sorting →
BANANAMAN	AMANBANAN
ANANAMANB	ANAMANBAN
NANAMANBA	ANANAMANB
ANAMANBAN	ANBANANAM
NAMANBANA	BANANAMAN
AMANBANAN	MANBANANA
MANBANANA	NAMANBANA
ANBANANAM	NANAMANBA
NBANANAMA	NBANANAMA

last column → NNBMNAAAA

index → index = 4

BWT: The Inverse Transform (Can we reconstruct the original message?)

index = 4

N
N
B
M
N
A
A
A
A

Given:

- Last column of sorted block "NNBMNAAAA"
- Index of original message in sorted block (4)

BWT: The Inverse Transform (Can we reconstruct the original message?)

index = 4

N
N
B
M
N
A
A
A
A

Given:

- Last column of sorted block "NNBMNAAAA"
- Index of original message in sorted block (4)

Decoding procedure

1. Create first column of sorted block (by sorting)

BWT: The Inverse Transform (Can we reconstruct the original message?)

index = 4

A	N
A	N
A	B
A	M
B	N
M	A
N	A
N	A
N	A

Given:

- Last column of sorted block "NNBMNAAAA"
- Index of original message in sorted block (4)

Decoding procedure

1. Create first column of sorted block (by sorting)

BWT: The Inverse Transform (Can we reconstruct the original message?)

index = 4

A	N
A	N
A	B
A	M
B	N
M	A
N	A
N	A
N	A

Given:

- Last column of sorted block "NNBMNAAAA"
- Index of original message in sorted block (4)

Decoding procedure

1. Create first column of sorted block (by sorting)
2. First symbol is given at transmitted index

BWT: The Inverse Transform (Can we reconstruct the original message?)

index = 4

	A	N
	A	N
	A	B
	A	M
4 →	B	N
	M	A
	N	A
	N	A
	N	A

decoded message:

B

Given:

- Last column of sorted block "N N B M N A A A A"
- Index of original message in sorted block (4)

Decoding procedure

1. Create first column of sorted block (by sorting)
2. First symbol is given at transmitted index

BWT: The Inverse Transform (Can we reconstruct the original message?)

index = 4

	A	N
	A	N
	A	B
	A	M
4 →	B	N
	M	A
	N	A
	N	A
	N	A

decoded message:

B

Given:

- Last column of sorted block "N N B M N A A A A"
- Index of original message in sorted block (4)

Decoding procedure

1. Create first column of sorted block (by sorting)
2. First symbol is given at transmitted index
3. Next symbol is obtained by

BWT: The Inverse Transform (Can we reconstruct the original message?)

index = 4

	A	N
	A	N
	A	B
	A	M
4 →	B	N
	M	A
	N	A
	N	A
	N	A

decoded message:

B

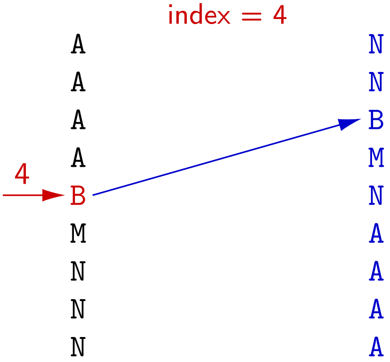
Given:

- Last column of sorted block "N N B M N A A A A"
- Index of original message in sorted block (4)

Decoding procedure

1. Create first column of sorted block (by sorting)
2. First symbol is given at transmitted index
3. Next symbol is obtained by
   a. Look for corresponding symbol in last column (i.e., same count of same letter)

BWT: The Inverse Transform (Can we reconstruct the original message?)



decoded message:

B

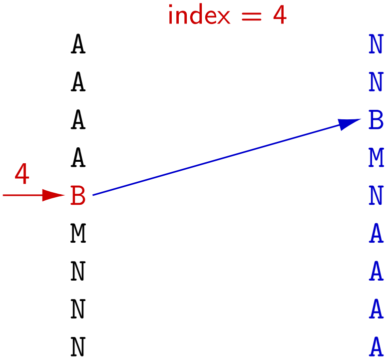
Given:

- Last column of sorted block "NNBMNAAAA"
- Index of original message in sorted block (4)

Decoding procedure

1. Create first column of sorted block (by sorting)
2. First symbol is given at transmitted index
3. Next symbol is obtained by
   a. Look for corresponding symbol in last column (i.e., same count of same letter)

BWT: The Inverse Transform (Can we reconstruct the original message?)



decoded message:

B

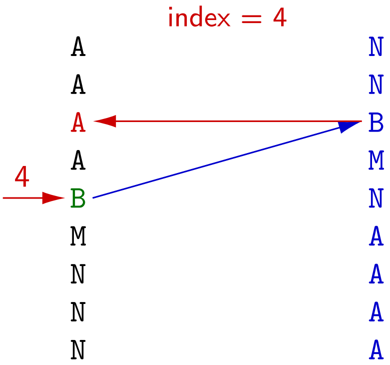
Given:

- Last column of sorted block "NNBMNAAAA"
- Index of original message in sorted block (4)

Decoding procedure

1. Create first column of sorted block (by sorting)
2. First symbol is given at transmitted index
3. Next symbol is obtained by
   a. Look for corresponding symbol in last column (i.e., same count of same letter)
   b. Next symbol is at same position in first column (since following symbol is in first column)

BWT: The Inverse Transform (Can we reconstruct the original message?)



decoded message:

B A

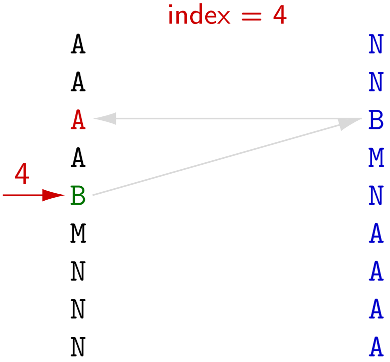
Given:

- Last column of sorted block “N N B M N A A A A”
- Index of original message in sorted block (4)

Decoding procedure

1. Create first column of sorted block (by sorting)
2. First symbol is given at transmitted index
3. Next symbol is obtained by
   a. Look for corresponding symbol in last column (i.e., same count of same letter)
   b. Next symbol is at same position in first column (since following symbol is in first column)

BWT: The Inverse Transform (Can we reconstruct the original message?)



decoded message:

B A

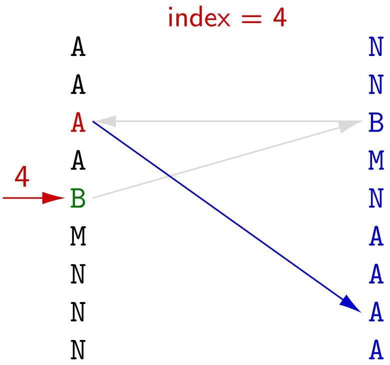
Given:

- Last column of sorted block "N N B M N A A A A"
- Index of original message in sorted block (4)

Decoding procedure

1. Create first column of sorted block (by sorting)
2. First symbol is given at transmitted index
3. Next symbol is obtained by
   - a. Look for corresponding symbol in last column (i.e., same count of same letter)
   - b. Next symbol is at same position in first column (since following symbol is in first column)
4. Continue procedure until all letters are decoded

BWT: The Inverse Transform (Can we reconstruct the original message?)



decoded message:

B A

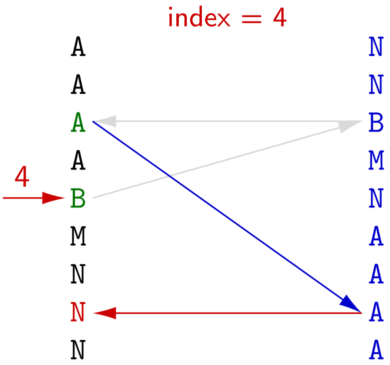
Given:

- Last column of sorted block "N N B M N A A A A"
- Index of original message in sorted block (4)

Decoding procedure

1. Create first column of sorted block (by sorting)
2. First symbol is given at transmitted index
3. Next symbol is obtained by
   - a) Look for corresponding symbol in last column (i.e., same count of same letter)
   - b) Next symbol is at same position in first column (since following symbol is in first column)
4. Continue procedure until all letters are decoded

BWT: The Inverse Transform (Can we reconstruct the original message?)



decoded message:

B A N

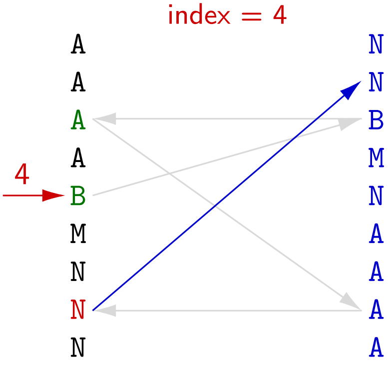
Given:

- Last column of sorted block "N N B M N A A A A"
- Index of original message in sorted block (4)

Decoding procedure

1. Create first column of sorted block (by sorting)
2. First symbol is given at transmitted index
3. Next symbol is obtained by
   - a. Look for corresponding symbol in last column (i.e., same count of same letter)
   - b. Next symbol is at same position in first column (since following symbol is in first column)
4. Continue procedure until all letters are decoded

BWT: The Inverse Transform (Can we reconstruct the original message?)



decoded message:

B A N

Given:

- Last column of sorted block "N N B M N A A A A"
- Index of original message in sorted block (4)

Decoding procedure

1. Create first column of sorted block (by sorting)
2. First symbol is given at transmitted index
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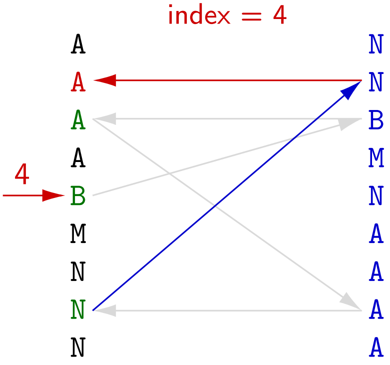
BWT: The Inverse Transform (Can we reconstruct the original message?)



decoded message:

B A N A

Given:

- Last column of sorted block “N N B M N A A A A”
- Index of original message in sorted block (4)

Decoding procedure

1. Create first column of sorted block (by sorting)
2. First symbol is given at transmitted index
3. Next symbol is obtained by
   - a. Look for corresponding symbol in last column (i.e., same count of same letter)
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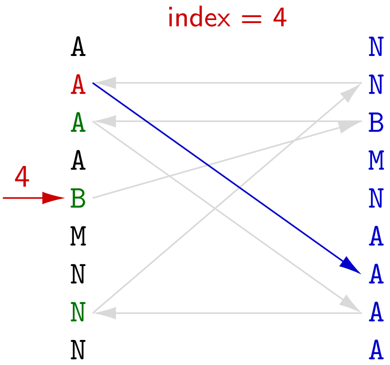

decoded message:

B A N A

Given:

- Last column of sorted block "N N B M N A A A A"
- Index of original message in sorted block (4)

Decoding procedure

1. Create first column of sorted block (by sorting)
2. First symbol is given at transmitted index
3. Next symbol is obtained by
   a. Look for corresponding symbol in last column (i.e., same count of same letter)
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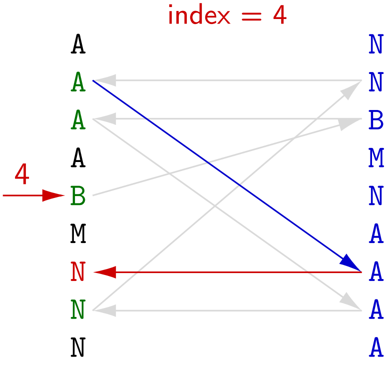
4. Continue procedure until all letters are decoded

BWT: The Inverse Transform (Can we reconstruct the original message?)



decoded message:

B A N A N

Given:

- Last column of sorted block "N N B M N A A A A"
- Index of original message in sorted block (4)

Decoding procedure

1. Create first column of sorted block (by sorting)
2. First symbol is given at transmitted index
3. Next symbol is obtained by
   a. Look for corresponding symbol in last column (i.e., same count of same letter)
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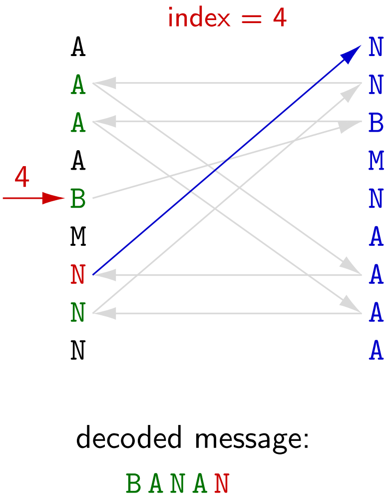
BWT: The Inverse Transform (Can we reconstruct the original message?)



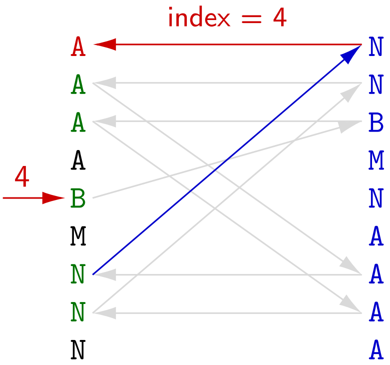
Given:

- Last column of sorted block "N N B M N A A A A"
- Index of original message in sorted block (4)

Decoding procedure

1. Create first column of sorted block (by sorting)
2. First symbol is given at transmitted index
3. Next symbol is obtained by
   a. Look for corresponding symbol in last column (i.e., same count of same letter)
   b. Next symbol is at same position in first column (since following symbol is in first column)
4. Continue procedure until all letters are decoded

BWT: The Inverse Transform (Can we reconstruct the original message?)



decoded message:

BANANA

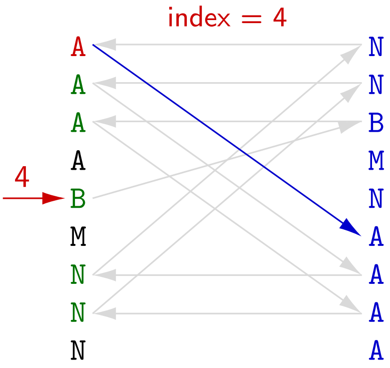
Given:

- Last column of sorted block "NNBMNAAAA"
- Index of original message in sorted block (4)

Decoding procedure

1. Create first column of sorted block (by sorting)
2. First symbol is given at transmitted index
3. Next symbol is obtained by
   - a. Look for corresponding symbol in last column (i.e., same count of same letter)
   - b. Next symbol is at same position in first column (since following symbol is in first column)
4. Continue procedure until all letters are decoded

BWT: The Inverse Transform (Can we reconstruct the original message?)



decoded message:

B A N A N A

Given:

- Last column of sorted block "N N B M N A A A A"
- Index of original message in sorted block (4)

Decoding procedure

1. Create first column of sorted block (by sorting)
2. First symbol is given at transmitted index
3. Next symbol is obtained by
   - a. Look for corresponding symbol in last column (i.e., same count of same letter)
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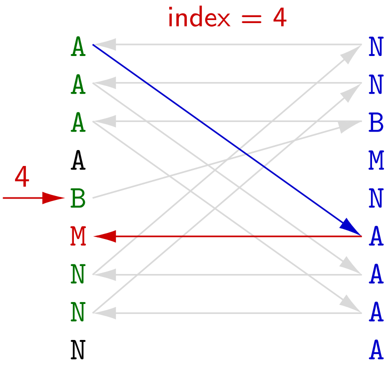
BWT: The Inverse Transform (Can we reconstruct the original message?)



decoded message:

B A N A N A M

Given:

- Last column of sorted block "N N B M N A A A A"
- Index of original message in sorted block (4)

Decoding procedure

1. Create first column of sorted block (by sorting)
2. First symbol is given at transmitted index
3. Next symbol is obtained by
   - a. Look for corresponding symbol in last column (i.e., same count of same letter)
   - b. Next symbol is at same position in first column (since following symbol is in first column)
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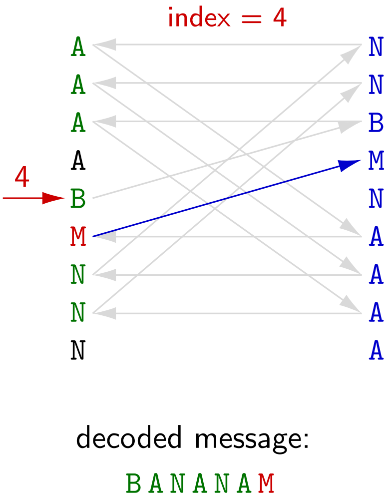
BWT: The Inverse Transform (Can we reconstruct the original message?)



Given:

- Last column of sorted block "N N B M N A A A A"
- Index of original message in sorted block (4)

Decoding procedure

1. Create first column of sorted block (by sorting)
2. First symbol is given at transmitted index
3. Next symbol is obtained by
   a. Look for corresponding symbol in last column (i.e., same count of same letter)
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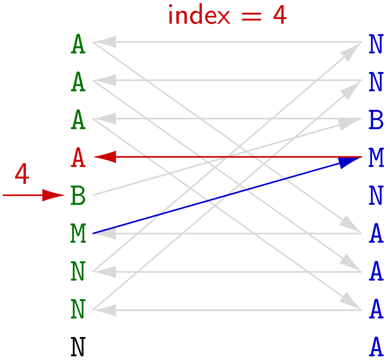

decoded message:

B A N A N A M A

Given:

- Last column of sorted block "N N B M N A A A A"
- Index of original message in sorted block (4)

Decoding procedure

1. Create first column of sorted block (by sorting)
2. First symbol is given at transmitted index
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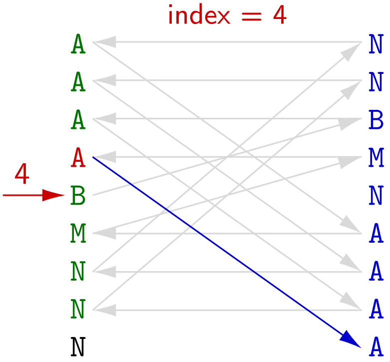

decoded message:

B A N A N A M A

Given:

- Last column of sorted block "N N B M N A A A A"
- Index of original message in sorted block (4)

Decoding procedure

1. Create first column of sorted block (by sorting)
2. First symbol is given at transmitted index
3. Next symbol is obtained by
   - a. Look for corresponding symbol in last column (i.e., same count of same letter)
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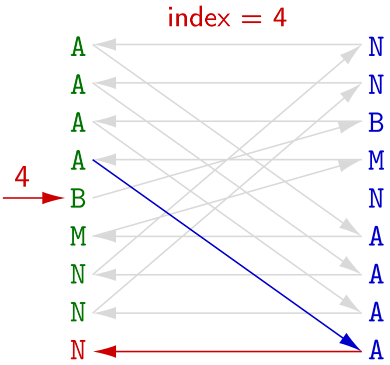
4. Continue procedure until all letters are decoded

BWT: The Inverse Transform (Can we reconstruct the original message?)



decoded message:

B A N A N A M A N

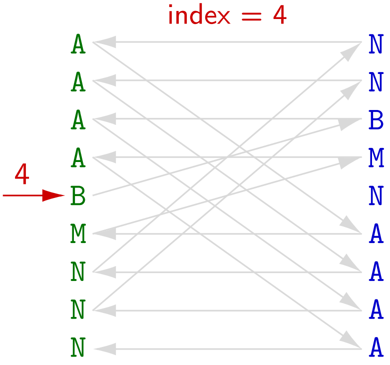
Given:

- Last column of sorted block "N N B M N A A A A"
- Index of original message in sorted block (4)

Decoding procedure

1. Create first column of sorted block (by sorting)
2. First symbol is given at transmitted index
3. Next symbol is obtained by
   a. Look for corresponding symbol in last column (i.e., same count of same letter)
   b. Next symbol is at same position in first column (since following symbol is in first column)
4. Continue procedure until all letters are decoded

BWT: The Inverse Transform (Can we reconstruct the original message?)



decoded message:

B A N A N A M A N

Given:

- Last column of sorted block "N N B M N A A A A"
- Index of original message in sorted block (4)

Decoding procedure

1. Create first column of sorted block (by sorting)
2. First symbol is given at transmitted index
3. Next symbol is obtained by
   a. Look for corresponding symbol in last column (i.e., same count of same letter)
   b. Next symbol is at same position in first column (since following symbol is in first column)
4. Continue procedure until all letters are decoded

BWT: Why Is It Useful for Compression ?

A M A N B A N A N
A N A M A N B A N
A N A N A M A N B
A N B A N A N A M
B A N A N A M A N
M A N B A N A N A
N A M A N B A N A
N A N A M A N B A
N B A N A N A M A

Property of BTW (for large blocks)

- Symbols on left side of sorted block are *contexts* (symbols that follow last column in message)

BWT: Why Is It Useful for Compression ?

```
A M A N B A N A N
A N A M A N B A N
A N A N A M A N B
A N B A N A N A M
B A N A N A M A N
M A N B A N A N A
N A M A N B A N A
N A N A M A N B A
N B A N A N A M A
```

Property of BTW (for large blocks)

- Symbols on left side of sorted block are *contexts* (symbols that follow last column in message)

BWT: Why Is It Useful for Compression ?

```
A M A N B A N A N
A N A M A N B A N
A N A N A M A N B
A N B A N A N A M
B A N A N A M A N
M A N B A N A N A
N A M A N B A N A
N A N A M A N B A
N B A N A N A M A
```

Property of BTW (for large blocks)

- Symbols on left side of sorted block are *contexts* (symbols that follow last column in message)
- Block lines are sorted according to the contexts

BWT: Why Is It Useful for Compression ?

```
A M A N B A N A N
A N A M A N B A N
A N A N A M A N B
A N B A N A N A M
B A N A N A M A N
M A N B A N A N A
N A M A N B A N A
N A N A M A N B A
N B A N A N A M A
```

Property of BTW (for large blocks)

- Symbols on left side of sorted block are *contexts* (symbols that follow last column in message)
- Block lines are sorted according to the contexts
- Likely that same symbol (last column) precedes same context (source with memory: conditional pmf with high peak)

BWT: Why Is It Useful for Compression ?

```
A M A N B A N A N
A N A M A N B A N
A N A N A M A N B
A N B A N A N A M
B A N A N A M A N
M A N B A N A N A
N A M A N B A N A
N A N A M A N B A
N B A N A N A M A
```

Property of BTW (for large blocks)

- Symbols on left side of sorted block are *contexts* (symbols that follow last column in message)
- Block lines are sorted according to the contexts
- Likely that same symbol (last column) precedes same context (source with memory: conditional pmf with high peak)

➡ **Last column contains long sequences of identical symbols**

BWT: Why Is It Useful for Compression ?

```
A M A N B A N A N
A N A M A N B A N
A N A N A M A N B
A N B A N A N A M
B A N A N A M A N
M A N B A N A N A
N A M A N B A N A
N A N A M A N B A
N B A N A N A M A
```

Property of BTW (for large blocks)

- Symbols on left side of sorted block are *contexts* (symbols that follow last column in message)
- Block lines are sorted according to the contexts
- Likely that same symbol (last column) precedes same context (source with memory: conditional pmf with high peak)

➡ **Last column contains long sequences of identical symbols**

How to exploit this property ?

- In following processing steps
- Example: Move-to-front transform (MTF)

The Move-To-Front Transform (MTF)

MTF: Map Symbols Sequences to Sequence of Unsigned Integers

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$$\mathcal{A} = \{\texttt{A B C D E F G H I J K L M N O P Q R S T U V W X Y Z}\}$$

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N N B M N A A A A $\qquad$ $\mathcal{A} = \{$A B C D E F G H I J K L M N O P Q R S T U V W X Y Z$\}$

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Example: Sequence "NNBMNAAAA" (result of BWT for "BANANAMAN")

N N B M N A A A A 13 $\mathcal{A} = \{$A B C D E F G H I J K L M N O P Q R S T U V W X Y Z$\}$

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N N B M N A A A A   13 $\mathcal{A} = \{$A B C D E F G H I J K L M N O P Q R S T U V W X Y Z$\}$

$\mathcal{A} = \{$N A B C D E F G H I J K L M O P Q R S T U V W X Y Z$\}$

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N N B M N A A A A	13	$\mathcal{A} = \{$A B C D E F G H I J K L M N O P Q R S T U V W X Y Z$\}$
N N B M N A A A A		$\mathcal{A} = \{$N A B C D E F G H I J K L M O P Q R S T U V W X Y Z$\}$

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N N B M N A A A A	0	$\mathcal{A} = \{$N A B C D E F G H I J K L M O P Q R S T U V W X Y Z$\}$
N N B M N A A A A	2	$\mathcal{A} = \{$N A B C D E F G H I J K L M O P Q R S T U V W X Y Z$\}$

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NNBMNAAAA	2	$\mathcal{A} = \{$N A B C D E F G H I J K L M O P Q R S T U V W X Y Z$\}$
NNBMNAAAA	13	$\mathcal{A} = \{$B N A C D E F G H I J K L M O P Q R S T U V W X Y Z$\}$
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NNBMNAAAA	2	$\mathcal{A} = \{$M B N A C D E F G H I J K L O P Q R S T U V W X Y Z$\}$
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N N B M N A A A A	2	$\mathcal{A} = \{$N A B C D E F G H I J K L M O P Q R S T U V W X Y Z$\}$
N N B M N A A A A	13	$\mathcal{A} = \{$B N A C D E F G H I J K L M O P Q R S T U V W X Y Z$\}$
N N B M N A A A A	2	$\mathcal{A} = \{$M B N A C D E F G H I J K L O P Q R S T U V W X Y Z$\}$
N N B M N A A A A	3	$\mathcal{A} = \{$N M B A C D E F G H I J K L O P Q R S T U V W X Y Z$\}$

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N **N** B M N A A A A	0	$\mathcal{A} = \{$N A B C D E F G H I J K L M O P Q R S T U V W X Y Z$\}$
N N **B** M N A A A A	2	$\mathcal{A} = \{$N A B C D E F G H I J K L M O P Q R S T U V W X Y Z$\}$
N N B **M** N A A A A	13	$\mathcal{A} = \{$B N A C D E F G H I J K L M O P Q R S T U V W X Y Z$\}$
N N B M **N** A A A A	2	$\mathcal{A} = \{$M B N A C D E F G H I J K L O P Q R S T U V W X Y Z$\}$
N N B M N **A** A A A	3	$\mathcal{A} = \{$N M B A C D E F G H I J K L O P Q R S T U V W X Y Z$\}$
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NNBMNAAAA	13	$\mathcal{A} = \{$A B C D E F G H I J K L M N O P Q R S T U V W X Y Z$\}$
N**N**BMNAAAA	0	$\mathcal{A} = \{$N A B C D E F G H I J K L M O P Q R S T U V W X Y Z$\}$
NN**B**MNAAAA	2	$\mathcal{A} = \{$N A B C D E F G H I J K L M O P Q R S T U V W X Y Z$\}$
NNB**M**NAAAA	13	$\mathcal{A} = \{$B N A C D E F G H I J K L M O P Q R S T U V W X Y Z$\}$
NNBM**N**AAAA	2	$\mathcal{A} = \{$M B N A C D E F G H I J K L O P Q R S T U V W X Y Z$\}$
NNBMN**A**AAA	3	$\mathcal{A} = \{$N M B A C D E F G H I J K L O P Q R S T U V W X Y Z$\}$
NNBMNA**A**AA	0	$\mathcal{A} = \{$A N M B C D E F G H I J K L O P Q R S T U V W X Y Z$\}$

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N **N** B M N A A A A	0	$\mathcal{A}$ = {N A B C D E F G H I J K L M O P Q R S T U V W X Y Z}
N N **B** M N A A A A	2	$\mathcal{A}$ = {N A B C D E F G H I J K L M O P Q R S T U V W X Y Z}
N N B **M** N A A A A	13	$\mathcal{A}$ = {B N A C D E F G H I J K L M O P Q R S T U V W X Y Z}
N N B M **N** A A A A	2	$\mathcal{A}$ = {M B N A C D E F G H I J K L O P Q R S T U V W X Y Z}
N N B M N **A** A A A	3	$\mathcal{A}$ = {N M B A C D E F G H I J K L O P Q R S T U V W X Y Z}
N N B M N A **A** A A	0	$\mathcal{A}$ = {A N M B C D E F G H I J K L O P Q R S T U V W X Y Z}
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N N **B** M N A A A A	2	$\mathcal{A} = \{$N A B C D E F G H I J K L M O P Q R S T U V W X Y Z$\}$
N N B **M** N A A A A	13	$\mathcal{A} = \{$B N A C D E F G H I J K L M O P Q R S T U V W X Y Z$\}$
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N N **B** M N A A A A	2	$\mathcal{A}$ = {N A B C D E F G H I J K L M O P Q R S T U V W X Y Z}
N N B **M** N A A A A	13	$\mathcal{A}$ = {B N A C D E F G H I J K L M O P Q R S T U V W X Y Z}
N N B M **N** A A A A	2	$\mathcal{A}$ = {M B N A C D E F G H I J K L O P Q R S T U V W X Y Z}
N N B M N **A** A A A	3	$\mathcal{A}$ = {N M B A C D E F G H I J K L O P Q R S T U V W X Y Z}
N N B M N A **A** A A	0	$\mathcal{A}$ = {A N M B C D E F G H I J K L O P Q R S T U V W X Y Z}
N N B M N A A **A** A	0	$\mathcal{A}$ = {A N M B C D E F G H I J K L O P Q R S T U V W X Y Z}
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N N B M N A A A A	13	$\mathcal{A} = \{$A B C D E F G H I J K L M N O P Q R S T U V W X Y Z$\}$
N **N** B M N A A A A	0	$\mathcal{A} = \{$N A B C D E F G H I J K L M O P Q R S T U V W X Y Z$\}$
N N **B** M N A A A A	2	$\mathcal{A} = \{$N A B C D E F G H I J K L M O P Q R S T U V W X Y Z$\}$
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The Move-To-Front Transform (MTF)

MTF: Map Symbols Sequences to Sequence of Unsigned Integers

1. Replace next symbol with its alphabet index
2. Update alphabet $\mathcal{A}$ by moving symbol to the front

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➡ **Effect**: Many small values for sequences with long repetitions (e.g., results of a BWT)

File Compression Utility BZIP2

Main Components for Compression

- Run-length encoding of input data (special V2V code)
- Block-wise **Burrows-Wheeler Transform** (BWT)
- **Move-To-Front Transform** (MTF) of BWT result
- Run-length encoding of MTF result
- Dynamic **Huffman coding**

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Some more details

- Block size for BWT/MTF of up to 900 kBytes
- Smart coding of Huffman tables
- Up to 6 Huffman tables per block
- Adaptive selection between Huffman tables (every 50 symbols)

Universal File Compressors

Marginal Huffman Coding

➡ Very old Unix utility **pack**

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Block Sorting: Burrows-Wheeler & Move-To-Front Transform

➡ File compressor **bzip2**

Lossless Audio Coding: Free Lossless Audio Codec (FLAC)

Basic Source Codec

1. Decompose audio file into variable-size blocks
   - → Block sizes determines capability for adaptation to signal statistics

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   a. No prediction
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4. Entropy coding of prediction error samples
   - Rice coding with adaptive Rice parameter selection

Lossless Image Coding: Portable Network Graphics (PNG)

Basic Source Codec

1. Separate Coding of Individual Color Planes

Lossless Image Coding: Portable Network Graphics (PNG)

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2. Prediction of Image Samples
   - ➡ Predictor is selected per image row
   - ➡ Five predictors are pre-defined (no adaptive prediction coefficients)

0	none	direct coding of image samples
1	left	prediction using left sample
2	above	prediction using above sample
3	average	prediction using rounded average of left and above sample
4	Paeth	non-linear prediction using left, above, and corner sample (**most often use**)

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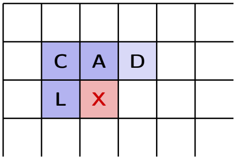
3. Entropy Coding of Prediction Error Samples
   - DEFLATE algorithm:
   - ➡ Lempel-Ziv-Storer-Szymanski (LZSS) algorithm for dependency removal
   - ➡ Huffman coding of LZSS output (adaptive Huffman tables)

Lossless Image Coding: JPEG-LS (Joint Photographic Experts Group)

Basic Source Codec

1. First prediction stage: LOCO Predictor

$$\hat{X} = \begin{cases} \min(L, A) & : \; C \geq \max(L, A) \\ \max(L, A) & : \; C \leq \min(L, A) \\ L + A - C & : \; \text{otherwise} \end{cases}$$



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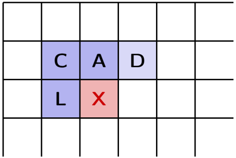

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2. Second order prediction using conditional mean $\mathrm{E}\{\, x \,|\, g_1, g_2, g_3 \,\}$
   - Given by clipped gradients (365 contexts after merging contexts with positive and negative signs)

$$\begin{aligned} g_1 &= \max(-4, \min(4, D - A)) \\ g_2 &= \max(-4, \min(4, A - C)) \\ g_3 &= \max(-4, \min(4, C - L)) \end{aligned}$$

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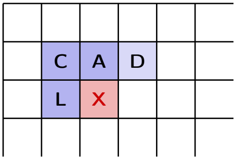

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3. Entropy Coding of Prediction Error Samples
   - Rice codes
   - Optional: Run-length coding (for uniform areas)

Comparison: Universal vs Specialized Compressors

		text	images	audio
				
compression		compression factor	compression factor	compression factor
gzip	(DEFLATE)	2.60	1.20	1.09
lzip	(LZMA)	3.53	1.41	1.17
bzip2	(BWT+MTF)	3.55	1.39	1.15
PNG	(prediction)		1.62	
FLAC	(prediction)			1.82

➡ **Specialized Compressors achieve Higher Coding Efficiency**

Summary of Lecture

Dictionary-based Coding

- Lempel-Ziv 1977 and 1978 algorithms (LZ77, LZ78): Basis for many universal compressors
- Lempel-Ziv-Storer-Szymanski (LZSS): Variant of LZ77
- Lempel-Ziv-Welch (LZW): Variant of LZ78
- DEFLATE: Combining LZSS with Huffman Coding
- Lempel-Ziv-Markov Chain Algorithm (LZMA): LZ78 Variant with Binary Arithmetic Coding

Lossless Coding using Block Sorting

- Burrows-Wheeler Transform (BWT)
- Move-To-Front Transform (MFT)

Lossless Compression Applications

- Universal File Compression: compress, gzip, bezip2, lzip
- Lossless Audio Coding: FLAC
- Lossless Image Coding: PNG, JPEG-LS

Exercise: Lossless Image Compression Challenge (Part II)

Improve your codec for lossless coding of 8-bit color images

- Try different things discussed in lectures and exercises

The following might be worth trying

- Prediction
  - Simple prediction using left sample
  - Fixed non-linear predictor like LOCO or Paeth predictor
  - Line- or block-adaptive selection of predictor (e.g., between horizontal, vertical, ...)
- Entropy Coding of Prediction Errors
  - Simple Rice codes (may be with adaptive Rice parameter)
  - Arithmetic coding with adaptive marginal pmf
  - Arithmetic coding with conditional pmf (very simple conditions)

Measure and provide the compressed file sizes for the Kodak test set!