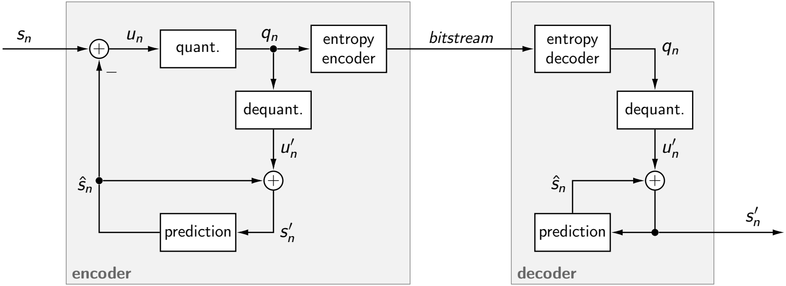
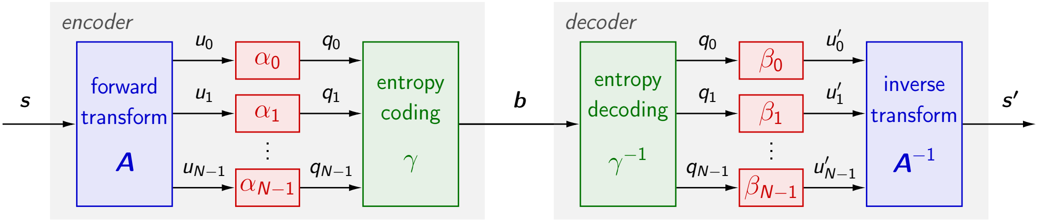
Predictive Quantization



Last Lectures: Transform Coding with Orthogonal Block Transform



- Forward transform: $\boldsymbol{u} = \boldsymbol{A} \cdot \boldsymbol{s}$
- Scalar quantization: $q_k = \alpha_k(u_k)$
- Entropy coding: $\boldsymbol{b} = \gamma(\{q_k\})$

- Entropy decoding: $\{q_k\} = \gamma^{-1}(\boldsymbol{b})$
- Dequantization: $u'_k = \beta_k(q_k)$
- Inverse transform: $\boldsymbol{s}' = \boldsymbol{A}^{-1} \cdot \boldsymbol{u}'$

Orthogonal Block Transforms

- Inverse matrix is equal to transposed matrix: $\boldsymbol{A}^{-1} = \boldsymbol{A}^{\mathrm{T}}$
- Geometric Interpretation: Rotation (and possible reflection) in N-d signal space
- ➞ Can be interpreted as lattice vector quantizer with orthogonal lattice
- Preservation of MSE distortion: Independent quantization of individual transform coefficients

Last Lectures: High-Rate Approximations

High-Rate Approximation of Operational Distortion-Rate Function

- Optimal bit allocation at high rates: $D_k(R_k) = \text{const}$ (for URQs: $\Delta_k = \text{const}$)

➡ High-rate distortion-rate function for transform coding (with optimal bit allocation)

$$D_{TC}(R) = \tilde{\varepsilon}^2 \cdot \tilde{\sigma}^2 \cdot 2^{-2R} \qquad \text{with} \qquad \tilde{\varepsilon}^2 = \left(\prod_k \varepsilon_k^2 \right)^{\frac{1}{N}}, \quad \tilde{\sigma}^2 = \left(\prod_k \sigma_k^2 \right)^{\frac{1}{N}}$$

Goal of Transform Selection

- Minimize distortion $D_{TC}(R)$ for given rate R ➡ Maximize transform coding gain G_T

$$G_T = \frac{D_{SQ}(R)}{D_{TC}(R)} = \frac{\varepsilon_S^2 \cdot \sigma_S^2}{\tilde{\varepsilon}^2 \cdot \tilde{\sigma}^2}$$

- Typically, ignore impact of pdf shapes ➡ **Maximize energy compaction G_{EC} of transform**

$$G_{EC} = \frac{\sigma_S^2}{\tilde{\sigma}^2} = \frac{\frac{1}{N} \sum_{k=0}^{N-1} \sigma_k^2}{\sqrt[N]{\prod_{k=0}^{N-1} \sigma_k^2}}$$

Last Lectures: Transform Selection

Karhunen Loève Transform (KLT)

- Basis vectors (rows of $\boldsymbol{A}$) are unit-norm eigenvectors of the N-th order auto-covariance matrix $\boldsymbol{C}_N$

➡ KLT yields uncorrelated transform coefficients ($\boldsymbol{C}_{UU}$ becomes a diagonal matrix)

➡ KLT achieves maximum possible energy compaction

$$G_{EC}^{(\mathrm{KLT})} = \left| \frac{1}{\sigma_S^2} \boldsymbol{C}_N \right|^{-\frac{1}{N}} \qquad \text{(note: determinant } |\cdot| = \text{product of eigenvalues)}$$

Discrete Cosine Transform of Type II (DCT-II)

- Represents discrete Fourier transform of mirrored signal (with half-sample symmetry of both sides)
- Signal independent: Basis vectors (rows of $\boldsymbol{A}$) are sampled cosines of different frequencies
- KLT approaches DCT-II for highly correlated sources ($\varrho \rightarrow 1$)
- For typical stationary signals: Only small loss in coding efficiency relative to KLT

➡ **Most widely used transform in image and video coding** (fast implementations possible)

Last Lectures: Coding Efficiency of Transform Coding

High-Rate Approximation for KLT and Gauss-Markov

- High-rate operational distortion-rate function and transform coding gain (for transform size N)

$$D_N(R) = \varepsilon^2 \cdot \sigma_S^2 \cdot (1-\varrho^2)^{\frac{N-1}{N}} \cdot 2^{-2R} \qquad \text{and} \qquad G_T^N = G_{EC}^N = (1-\varrho^2)^{\frac{1-N}{N}}$$

➡ Transform gain increases with transform size N, but approaches a limit

Comparison to Rate-Distortion Bound

- Example: Gauss-Markov, KLT, and optimal scalar quantizers (ECSQ)

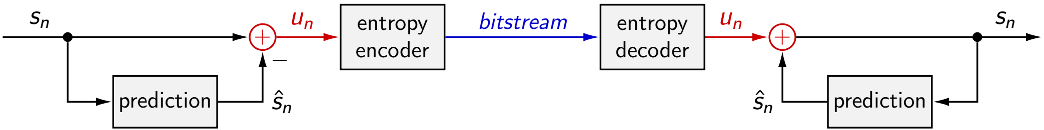
➡ Distortion increase relative to Shannon lower bound

$$\frac{D_N^{(\mathrm{KLT})}(R)}{D_{\mathrm{SLB}}(R)} = \frac{\frac{\pi e}{6} \cdot \sigma_S^2 \cdot (1-\varrho^2)^{\frac{N-1}{N}} \cdot 2^{-2R}}{\sigma_S^2 \cdot (1-\varrho^2) \cdot 2^{-2R}} = \frac{\pi e}{6} \cdot \left(\frac{1}{1-\varrho^2}\right)^{\frac{1}{N}}$$

➡ Large transform sizes ($N \to \infty$): Performance gap reduces to space-filling gain (1.53 dB)

- Other sources: Transform coding cannot utilize all dependencies (non-linear dependencies)

Review: Lossless Coding using Prediction



Predictive Lossless Coding

- Predict current sample s_n using a function of preceding samples (referred to as observation set $\boldsymbol{b}_n$)

$$\hat{s}_n = f(s_{n-1}, s_{n-2}, \cdots) = f(\boldsymbol{b}_n)$$

- Entropy coding of prediction error samples

$$u_n = s_n - \hat{s}_n$$

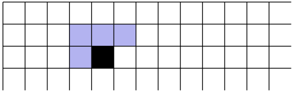
- Decoder reconstructs the original samples according to

$$s_n = \hat{s}_n + u_n$$

examples of observation sets

1D signals:

2D signals:



Review: Linear and Affine Prediction

Linear Predictor

- Current sample is predicted using weighted sum over observation set $\boldsymbol{b}_n = \{b_1, \cdots, b_n\}$

$$\hat{s}_n = \sum_{k=1}^{N} a_k \cdot b_k = \boldsymbol{a}^{\mathrm{T}} \cdot \boldsymbol{b}_n \qquad \text{with} \qquad \boldsymbol{a} = \left(a_1, \cdots, a_N\right)^{\mathrm{T}}$$

➡ **Prediction error variance** σ_U^2 is given by

$$\sigma_U^2(\boldsymbol{a}) = \sigma_S^2 - 2\boldsymbol{a}^{\mathrm{T}}\boldsymbol{c} + \boldsymbol{a}^{\mathrm{T}}\boldsymbol{C}_B\boldsymbol{a} \qquad \text{with} \qquad \begin{aligned} \boldsymbol{C}_B &= \mathrm{E}\left\{ \left(\boldsymbol{B}_n - \mathrm{E}\{\boldsymbol{B}_n\}\right)\left(\boldsymbol{B}_n - \mathrm{E}\{\boldsymbol{B}_n\}\right)^{\mathrm{T}} \right\} \\ \boldsymbol{c} &= \mathrm{E}\left\{ \left(S_n - \mathrm{E}\{S_n\}\right)\left(\boldsymbol{B}_n - \mathrm{E}\{\boldsymbol{B}_n\}\right) \right\} \end{aligned}$$

➡ Prediction error variance is minimized by solution of the **Yule-Walker equations**

$$\boldsymbol{C}_B \cdot \boldsymbol{a}_{\mathrm{opt}} = \boldsymbol{c} \qquad \text{(linear equation system)}$$

➡ Prediction error variance for optimal linear prediction

$$\sigma_U^2 \;=\; \sigma_S^2 - \boldsymbol{a}_{\mathrm{opt}}^{\mathrm{T}}\boldsymbol{c} \;=\; \sigma_S^2 - \boldsymbol{c}^{\mathrm{T}}\boldsymbol{C}_B^{-1}\boldsymbol{c}$$

Review: Linear and Affine Prediction

Affine Predictor

- Linear predictor with additional constant offset

$$\hat{s}_n = a_0 + \sum_{k=1}^{N} a_k \cdot b_k = a_0 + \boldsymbol{a}^{\mathrm{T}} \cdot \boldsymbol{b}_n$$

➡ Prediction error variance σ_U^2 is not impacted by constant offset a_0

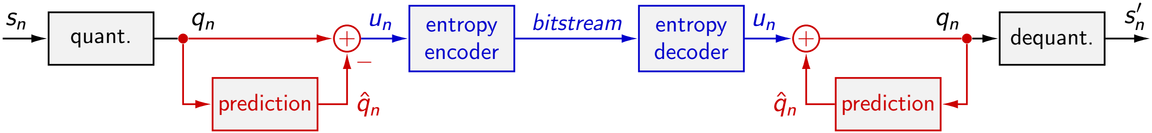
➡ Mean of prediction error μ_U can be forced to zero by choosing

$$a_0 = \mu_S \left(1 - \sum_{k=1}^{N} a_k \right)$$

➡ Affine prediction reduces mean squared prediction error (compared to linear prediction)

$$\varepsilon_U^2 = \mathrm{E}\left\{ \left(S_n - \hat{S}_n \right)^2 \right\} = \sigma_U^2 + \mu_U^2$$

Lossy Coding with Prediction: (1) Prediction of Quantization Indexes



Encoder

- Scalar quantization

$$q_n = \alpha(s_n)$$

- Prediction of quantization indexes

$$\hat{q}_n = f_{\text{pred}}(q_{n-1}, q_{n-2}, \cdots)$$

- Determining prediction residual

$$u_n = q_n - \hat{q}_n$$

- Entropy coding of prediction residual

Decoder

- Entropy decoding of prediction residual
- Prediction of quantization indexes

$$\hat{q}_n = f_{\text{pred}}(q_{n-1}, q_{n-2}, \cdots)$$

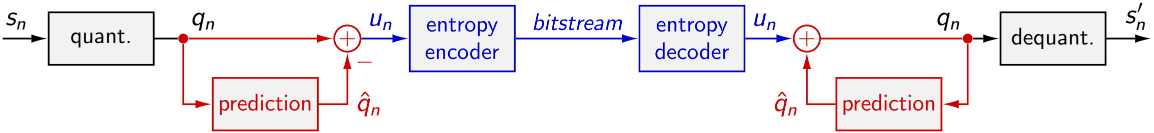
- Reconstruct quantization indexes

$$q_n = u_n + \hat{q}_n$$

- Dequantization (typically: scaling)

$$s'_n = \beta(q_n)$$

Lossy Coding with Prediction: (1) Prediction of Quantization Indexes



Prediction after Quantization

- Sources with memory: Quantization indexes have statistical dependencies

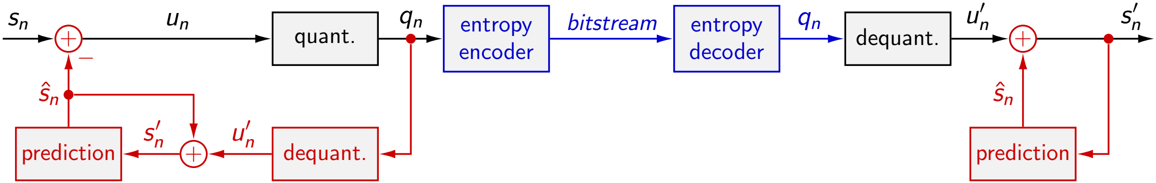
➡ Prediction can improve lossless coding of quantization indexes

  - More accurate: With prediction, entropy coding can be simplified
  - Require: Predicted value $\hat{q}_n$ must be integer (since u_n must be integer)

Extension: Prediction after Transform and Quantization

- Prediction of some transform coefficients (e.g., DC coefficient)
- Examples: JPEG, MPEG-2 Video, H.263, MPEG-4 Visual

Lossy Coding with Prediction: (2) Quantization of Prediction Error



Prediction Before Quantization

- Idea: Reduce statistical dependencies before quantization (similar to transform coding)
- Have to use same prediction signal $\hat{s}_n$ at encoder and decoder (otherwise: error accumulation)

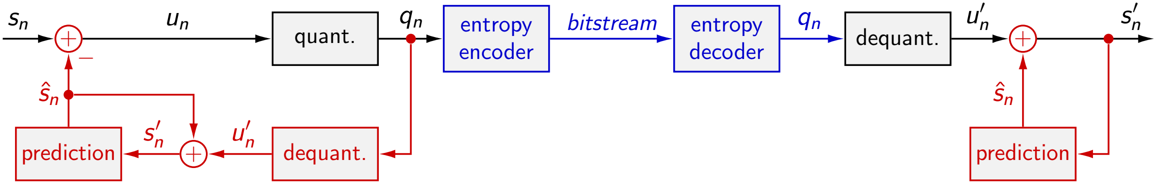
➡ Prediction value $\hat{s}_n$ has to be derived based on reconstructed samples

$$\hat{s}_n = f_{\text{pred}}\left(s'_{n-1}, s'_{n-2}, \cdots\right)$$

➡ Encoder includes decoder (except for entropy decoding)

- Concept also referred to as **differential pulse code modulation (DPCM)**

Lossy Coding with Prediction: (2) Quantization of Prediction Error



DPCM Encoder

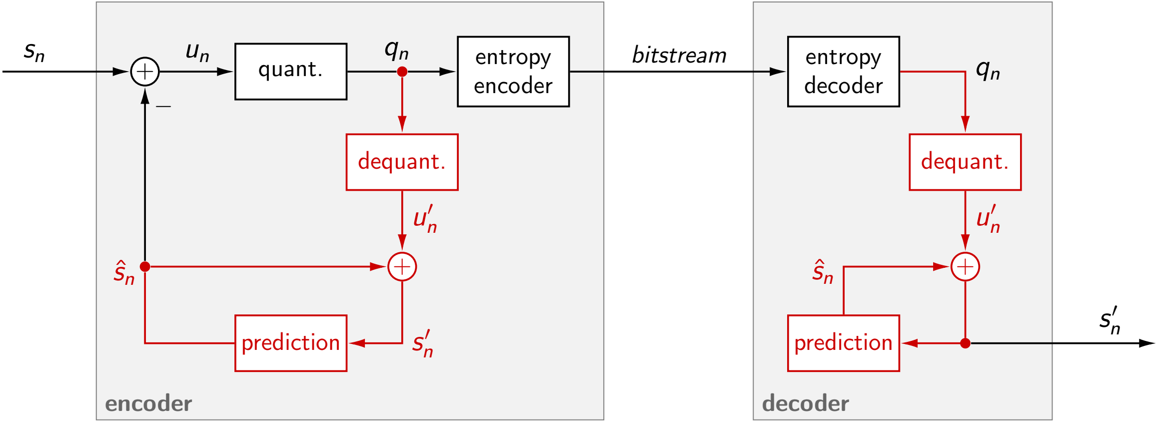
- Prediction: $\hat{s}_n = f_{\text{pred}}(s'_{n-1}, s'_{n-2}, \cdots)$
- Quantization: $q_n = \alpha(s_n - \hat{s}_n)$
- Reconstruction: $s'_n = \hat{s}_n + \beta(q_n)$
- Entropy coding of quantization indexes q_n

DPCM Decoder

- Entropy decoding of quantization indexes q_n
- Prediction: $\hat{s}_n = f_{\text{pred}}(s'_{n-1}, s'_{n-2}, \cdots)$
- Reconstruction: $s'_n = \hat{s}_n + \beta(q_n)$

DPCM Encoder and Decoder

- Redrawing yields typical DPCM structure
- Note: Encoder contains decoder except entropy decoding



Design of Optimal Predictor and Quantizer

Interdependencies between Predictor and Quantizer

- Optimal quantizer depends on statistical properties of prediction error signal
- Optimal predictor depends on statistical properties of quantization error

Joint Design of Predictor and Quantizer

- Choose sufficiently large training set and Lagrange multiplier
- Start: Design predictor for original training data
- Iteratively refine quantizer and predictor
  - ➡ Design quantizer for predictor error signal (using current predictor)
  - ➡ Run DPCM encoding with current predictor and quantizer
  - ➡ Update predictor using obtained reconstructed signal

Simplified Design

- Design predictor for original signal (i.e., neglect quantization in predictor design)
- Typically: Works reasonably well

Example: DPCM for Gauss-Markov

Gauss-Markov Model

- Zero-mean AR(1): Ignore mean, since it can be removed as a first step

$$S_n = \varrho \cdot S_{n-1} + Z_n \qquad \text{(dependencies are specified by correlation coefficient } \varrho\text{)}$$

- Innovation process Z_n is zero-mean Gaussian iid

DPCM Coding of Gauss-Markov Source

- Consider optimal linear predictor for original source (i.e., one-tap filter $a = \varrho$)

$$\hat{s}_n = \varrho \cdot s'_{n-1} = \varrho \cdot (s_{n-1} - x_{n-1}) \qquad \text{with quantization error} \qquad x_n = u_n - u'_n = s_n - s'_n$$

➞ Resulting prediction error

$$\begin{aligned} u_n &= s_n - \hat{s}_n \\ &= s_n - \varrho \cdot s_{n-1} + \varrho \cdot x_{n-1} \end{aligned}$$

DPCM for Gauss-Markov: Prediction Error Variance

➡ **Variance of prediction error** $u_n = s_n - \varrho \cdot s_{n-1} + \varrho \cdot x_{n-1}$ with $x_n = s_n - s'_n$

$$
\begin{aligned}
\sigma_U^2 &= \mathrm{E}\{\, U_n^2 \,\} \\
&= \mathrm{E}\{\, (S_n - \varrho S_{n-1} + \varrho X_{n-1})^2 \,\} \\
&= \mathrm{E}\{\, S_n^2 \,\} + \varrho^2 \cdot \mathrm{E}\{\, S_{n-1}^2 \,\} - 2\varrho \cdot \mathrm{E}\{\, S_n S_{n-1} \,\} + \varrho^2 \cdot \mathrm{E}\{\, X_{n-1}^2 \,\} \\
&\qquad + 2\varrho \cdot \mathrm{E}\{\, S_n X_{n-1} \,\} - 2\varrho^2 \cdot \mathrm{E}\{\, S_{n-1} X_{n-1} \,\} \\
&= \sigma_S^2 + \varrho^2 \sigma_S^2 - 2\varrho^2 \sigma_S^2 + \varrho^2 D \qquad\qquad (\text{note: distortion } D = \mathrm{E}\{\, (S_n - S'_n)^2 \,\}) \\
&\qquad + 2\varrho \mathrm{E}\{\, (\varrho S_{n-1} + Z_n) E_{n-1} \,\} - 2\varrho^2 \mathrm{E}\{\, S_{n-1} E_{n-1} \,\} \\
&= \sigma_S^2 (1 - \varrho^2) + \varrho^2 D + 2\varrho^2 \mathrm{E}\{\, S_{n-1} E_{n-1} \,\} - 2\varrho^2 \mathrm{E}\{\, S_{n-1} E_{n-1} \,\} + 2\varrho \mathrm{E}\{\, Z_n E_{n-1} \,\} \\
&= \sigma_S^2 (1 - \varrho^2) + \varrho^2 D + 2\varrho \mathrm{E}\{\, Z_n E_{n-1} \,\} \\
&= \sigma_S^2 (1 - \varrho^2) + \varrho^2 D
\end{aligned}
$$

DPCM for Gauss-Markov: Prediction Error Variance

Prediction Error Variance for Predictor $\hat{s}_n = \varrho \cdot s'_n$

➡ Prediction error variance σ_U^2 depends on distortion D

$$\sigma_U^2 \;=\; \sigma_S^2\,(1-\varrho^2) + \varrho^2 D$$

➡ Distortion D is caused by scalar quantization of prediction error u_n

$$D(R) = \sigma_U^2 \cdot g(R)$$

where $g(R)$ is the distortion-rate function for a unit-variance Gaussian

➡ **Rate-dependent prediction error variance**

$$\sigma_U^2 \;=\; \sigma_S^2\,(1-\varrho^2) + \varrho^2 \cdot \sigma_U^2 \cdot g(R)$$

$$\Rightarrow \quad \boxed{\sigma_U^2 \;=\; \sigma_S^2 \cdot \frac{1-\varrho^2}{1-\varrho^2 \cdot g(R)}}$$

DPCM for Gauss-Markov: Distortion-Rate Function

Distortion-Rate Function

- Approximation of operational distortion-rate function

$$D(R) \;=\; \sigma_U^2 \cdot g(R) \;=\; \sigma_S^2 \cdot \frac{1-\varrho^2}{1-\varrho^2 \cdot g(R)} \cdot g(R)$$

High-Rate Approximation

- High rates R: $\quad g(R) = \varepsilon^2 \cdot 2^{-2R} \quad$ and $\quad g(R) \ll 1$

➡ Operational distortion-rate function at high rates

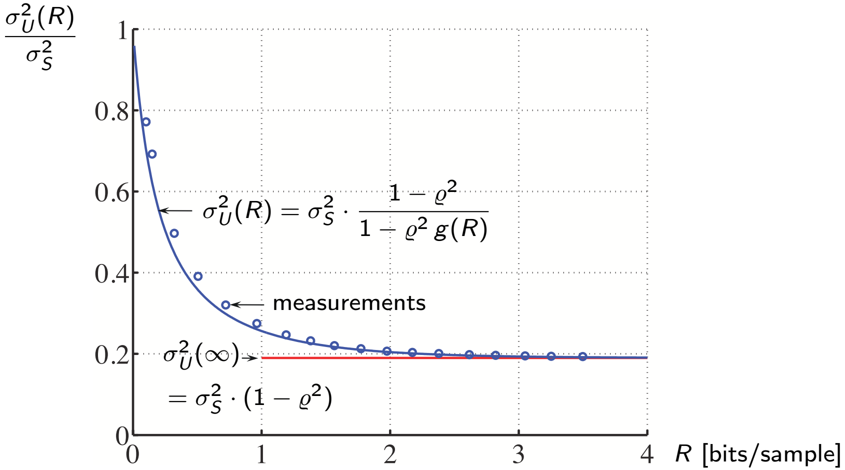
$$D(R) = \varepsilon^2 \cdot \sigma_S^2 \cdot (1-\varrho^2) \cdot 2^{-2R}$$

➡ Same formula as for transform coding with large KLT $\quad (N \to \infty)$

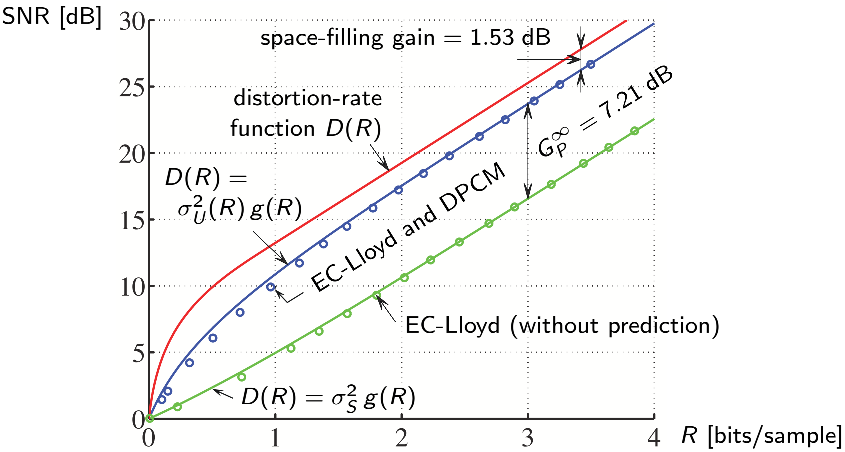
➡ Gap to rate-distortion bound represents space-filling gain of vector quantization

DPCM for Gauss-Markov: Experimental Results for $\varrho = 0.9$

➡ Prediction error variance σ_U^2 depends on bit rate



DPCM for Gauss-Markov: Experimental Results for $\varrho = 0.9$



- High rates: Shape and memory gain are achievable
- Low rates: Worse than transform coding

Adaptive DPCM

Audio signals, images, videos

- Instationary signals
- Single predictor not suitable for entire signal
- Adapt predictor (incl. observation set) during encoding/decoding

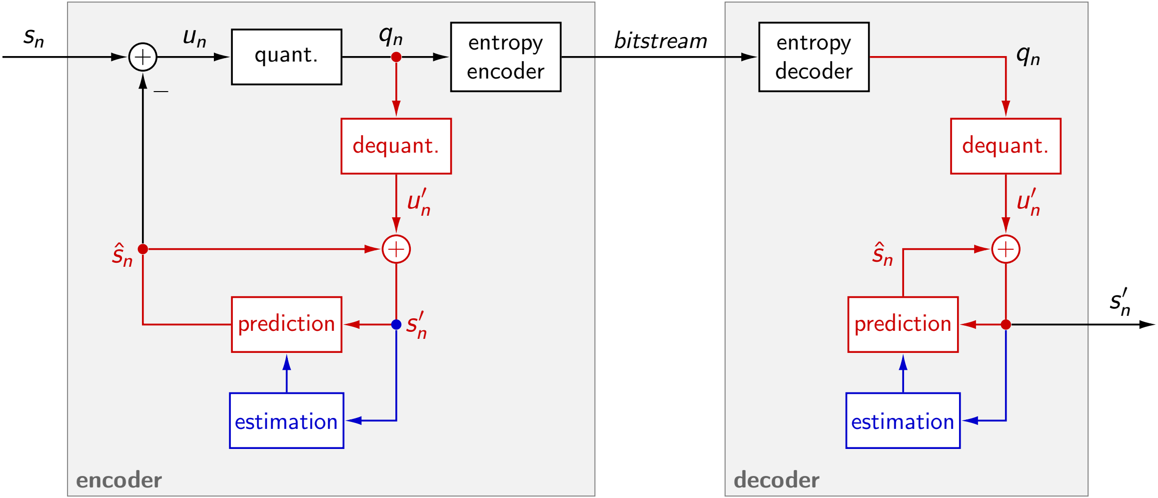
Backward adaptation

- Simultaneously estimate predictor at encoder and decoder side
- Estimation has to be based on reconstructed samples
- No additional rate, but accuracy, complexity, error resilience issues

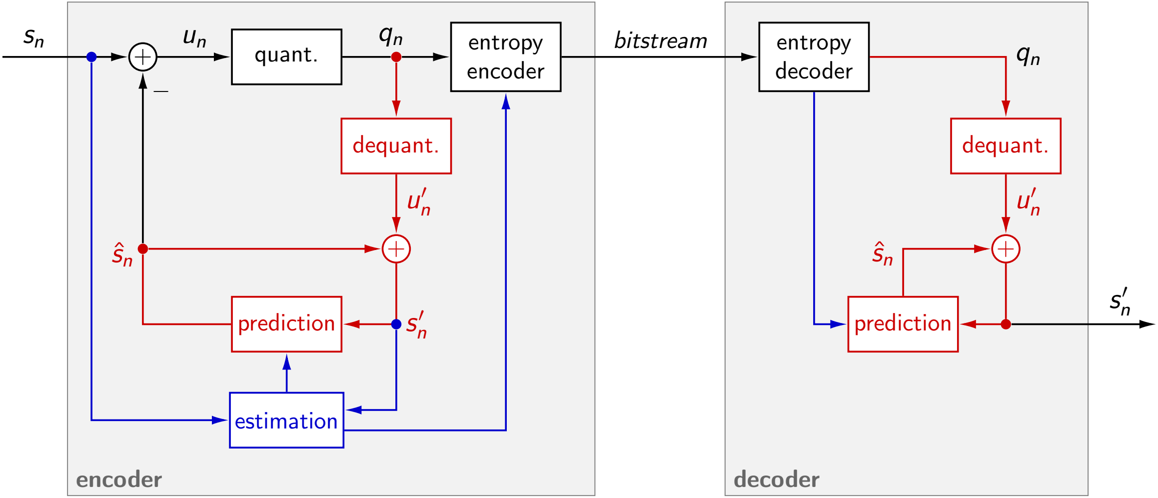
Forward adaptation

- Analyze signal at encoder side
- Send predictor as part of the bitstream (requires additional bit rate)
- Simple method: Switched predictors

Backward-Adaptive DPCM



Forward-Adaptive DPCM



Lossy Source Coding: DPCM or Transform Coding ?

Differential Pulse Coding Modulation (DPCM)

- Worse coding efficiency than transform coding at low rates (interesting opertation points)
- Audio, images, video: Perceptual artifacts when using rather strong quantization

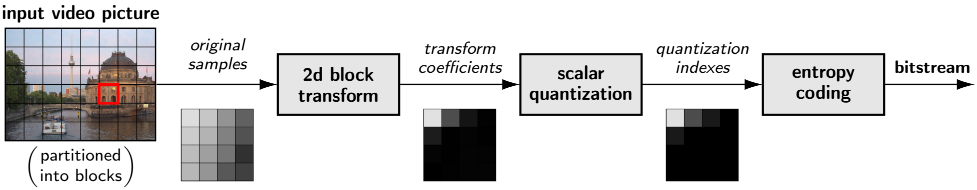
Transform Coding

- Better coding efficiency than DPCM at low rates (interesting operation points)
- Better subjective quality than DPCM: Can better control perceived quality (frequency spectrum)
- Cannot utilize statistical dependencies between blocks (only inside blocks)

Combination of Transform Coding and Prediction

- Improve coding efficiency of transform coding by prediction between transform blocks
- Two concepts:
  1. Prediction after quantization: Only for certain transform coefficients
  2. Prediction before transform: DPCM with transform coding as quantizer

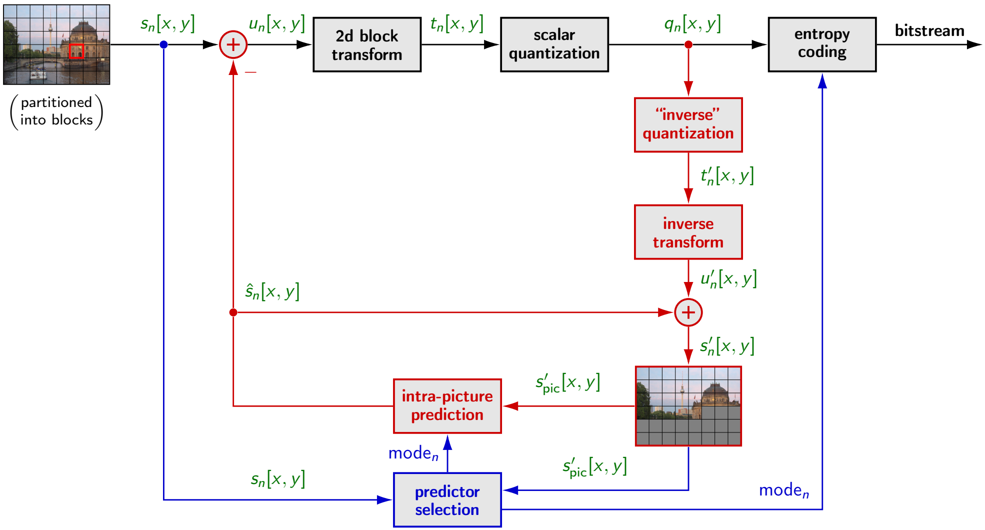
Image Coding Example: JPEG



Block-based Transform Coding

1. **Transform**: Separable DCT-II for 8×8 blocks of samples
2. **Quantization**: Uniform reconstruction quantizer
3. **Entropy Coding**: DC Prediction and Run-Level Coding with Huffman tables
   - a. Prediction of quantization index for DC coefficient (mean of block): $\Delta q^n_{(0,0)} = q^n_{(0,0)} - q^{n-1}_{(0,0)}$
   - b. Huffman table for prediction error $\Delta q^n_{(0,0)}$ for DC coefficient
   - c. Run-level coding of remaining quantization indexes (without prediction)

Modern Image Coding: Forward-Adaptive DPCM Structure



Block-Based Intra-Picture Prediction

Block-Based Prediction and Transform Coding

- Predict signal of current block using reconstructed neighboring blocks
- Transform coding of prediction error signal

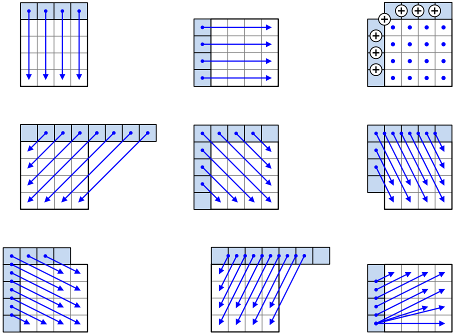
Intra-Picture Prediction

- Using surrounding reconstructed samples for predicting signal of current block
- Multiple modes with directional prediction

Examples

- H.264 | AVC: 9 prediction modes
- H.265 | HEVC: 35 prediction modes

intra prediction modes in H.264 | AVC



Successive Pictures in Video Sequences

successive picture are typically very similar

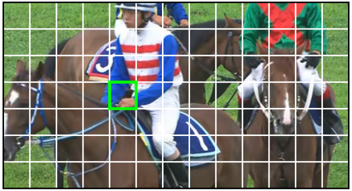
(exception: scene cuts)

Important: Utilize large amount of dependencies between video picture

➡ **Basic Idea**: Predict current picture from already coded previous picture

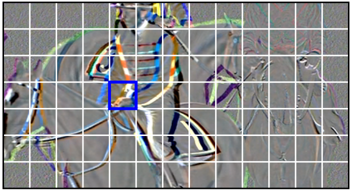
Simple Variant: Frame Difference Coding

rec. previous picture s'_{n-1}



current original picture s_n

prediction error u_n (FD)



- Partition current picture s_n into rectangular blocks
- Block-wise coding of current picture s_n
  1. Get prediction error: $u_n[x,y] = s_n[x,y] - s'_{n-1}[x,y]$
  2. Transform coding: $u_n[x,y] \mapsto u'_n[x,y]$
  3. Transmit in bitstream: Quantization indexes $\{q_k\}$
  4. Reconstruction: $s'_n[x,y] = s'_{n-1}[x,y] + u'_n[x,y]$

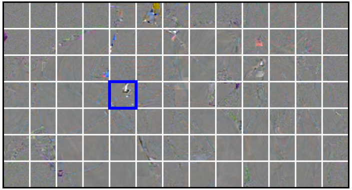
➡ **Problem: Ineffective for moving regions**

Improvement: Motion-Compensated Prediction

rec. previous picture s'_{n-1}

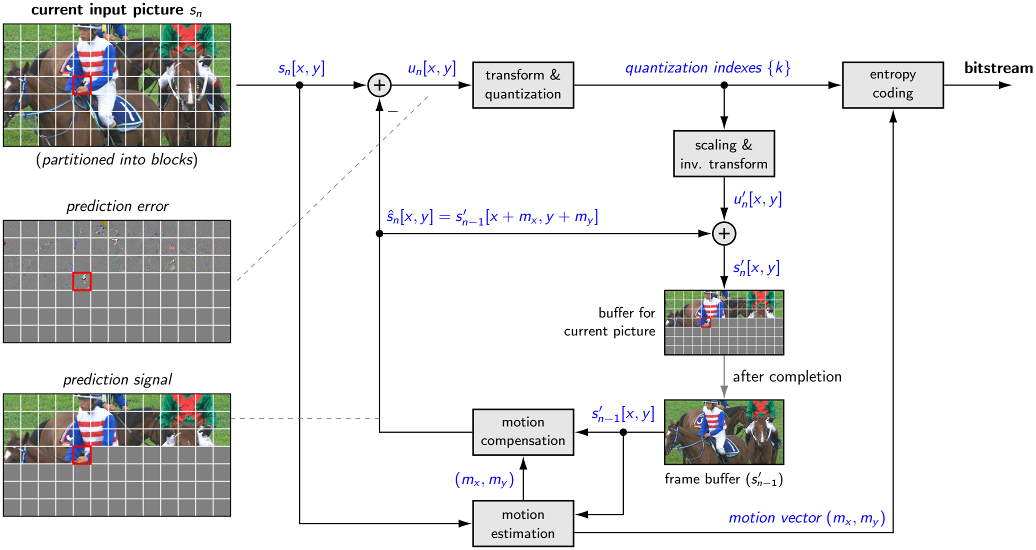
current original picture s_n

prediction error u_n (MCP)



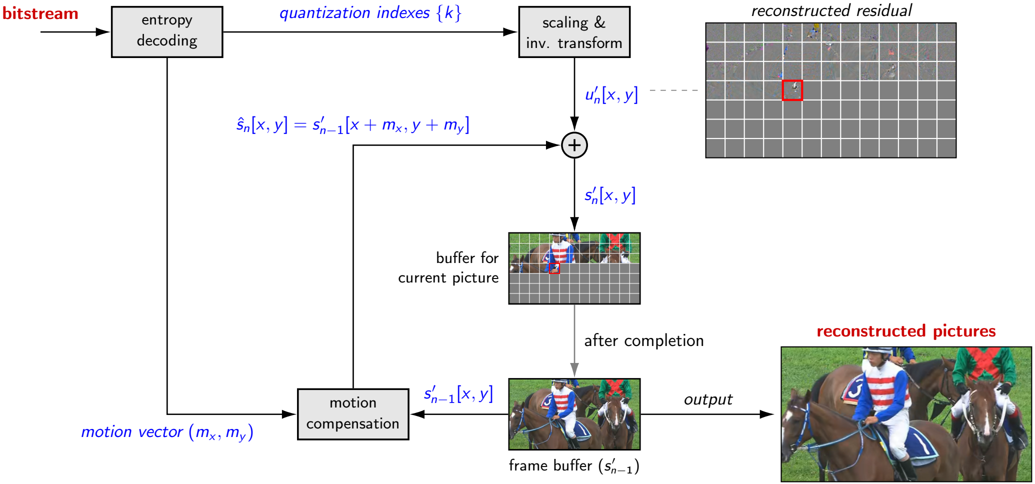
- Partition current picture s_n into rectangular blocks
- Estimate motion vectors (m_x, m_y) of blocks in current picture relative to previous picture s'_{n-1}
- Block-wise coding of current picture s_n
  1. Get prediction error: $u_n[x,y] = s_n[x,y] - s'_{n-1}[x+m_x, y+m_y]$
  2. Transform coding: $u_n[x,y] \mapsto u'_n[x,y]$
  3. Transmit in bitstream: Motion vector (m_x, m_y) and quantization indexes $\{q_k\}$
  4. Reconstruction: $s'_n[x,y] = s'_{n-1}[x+m_x, y+m_y] + u'_n[x,y]$

Hybrid Video Coding: Encoder

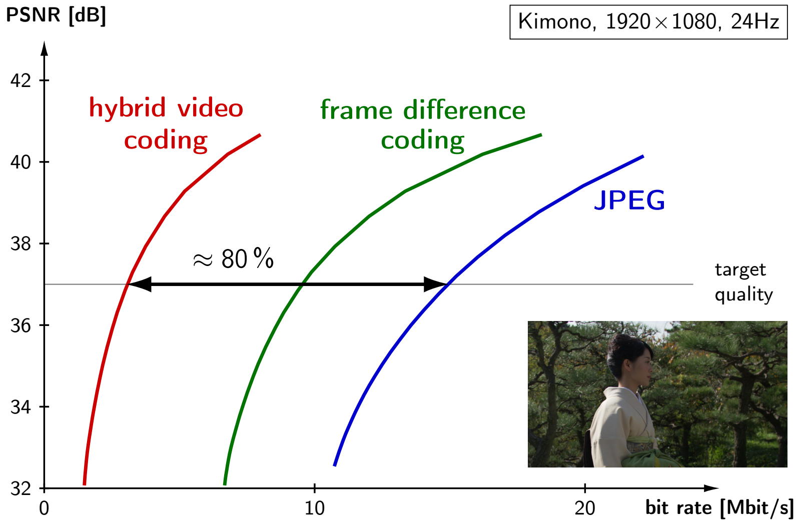


Hybrid Video Coding: Decoder

decoding of n-th picture



Efficiency of Hybrid Video Coding



Summary of Lecture

Prediction after Quantization

- Prediction of quantization indexes can improve lossless coding
- Combination with transform coding: Only useful for certain transform coefficients

Prediction before Quantization: Differential Pulse Code Modulation (DPCM)

- Reduce variance (or statistical dependencies) before quantization
- Have to use reconstructed samples for prediction in order to avoid error acculumation
- Quantization impacts quality of prediction (worse at low bit rates)
- Combination with transform coding: Prediction of complete blocks of samples

Image and Video Coding

- Combination of block-based prediction and transform coding
- Prediction of blocks of samples: Intra-picture prediction or motion-compensated prediction
- Transform coding of prediction error blocks

Exercise 1: Compare Your Lossy Codec to JPEG

Evaluate the Coding Efficiency of JPEG

- Choose a PPM image from our data base
- Encoding: Convert image to JPEG using different quality parameters (e.g., using ImageMagick)
- Decoding: Convert the JPEG file back to PPM format (e.g., using ImageMagick)
- Measure the RGB-PSNR between original and reconstructed image (tool available in KVV)
- Measure the bit rate (in bits per sample) based on size of the JPEG file

Evaluate the Coding Efficiency of your Codec

- Encode and decode the PPM image (same as for JPEG) with varying quantization step sizes
- Measure the bit rate of the compressed file and the RGB-PSNR of the reconstructed image

Compare Coding Efficiency of your Codec with that of JPEG

- Plot the RGB-PSNR over the bit rate for both your codec and JPEG (for multiple operation points)
- Compare your codec and JPEG by plotting the PSNR-rate curves into one diagram

Exercise 2: Lossy Image Compression Challenge

Improve your codec for lossy coding of PPM images

- Use any implementation of last weeks exercise as basis (see KVV)
- Try different simple techniques discussed in lectures and exercises

The following might be worth trying

- Use YCoCg format for actual coding (see implementations for lossless coding)
- Add prediction between transform blocks:
  - ➡ Prediction of quantization index for DC coefficient (as in JPEG); or
  - ➡ Subtract mean of surrounding reconstructed samples before transform (should be better), add that mean after reconstruction (dequantization + inverse transform) of prediction error
- Improve entropy coding of quantization indexes:
  - ➡ Adaptive arithmetic coding (see implementations for lossless coding)
  - ➡ Adaptive binary arithmetic coding (see implementations for lossless coding)