Exercise 1: Implement Lloyd Algorithm

Implement the Lloyd algorithm using a programming language of your choice.

- Test the algorithm (for quantizer sizes of $K = 2, 4, 8, 16, 32$) for
  - a unit-variance Gaussian pdf:
    $$f(s) = \frac{1}{\sqrt{2\pi}}\, e^{-\frac{1}{2}\, s^2}$$
  - a unit-variance Laplacian pdf:
    $$f(s) = \frac{1}{\sqrt{2}}\, e^{-\sqrt{2}\,|s|}$$
- Determine the distortion D for your quantizers.
- Compare the R-D performance of your quantizers (for $K = 2, 4, 8, 16, 32$) to the high-rate approximation for Lloyd quantizers with fixed-length codes.

You can implement the Lloyd algorithm that directly uses the pdf or the Lloyd algorithm that uses a training set (files with 1 000 000 samples in float32 format are provided on the course web site)

Exercise 2: Lloyd Quantizer for MSE Distortion (Alternative)

Given is a stationary source with a zero-mean Laplace pdf $f(x)$ and a symmetric 3-interval quantizer:

$$f(x) = \frac{1}{\sqrt{2\sigma^2}}\, e^{-\sqrt{\frac{2}{\sigma^2}}\,|x|} \qquad \text{and} \qquad Q(x) = \begin{cases} -b & : \quad x < -a \\ 0 & : \quad |x| \leq a \\ b & : \quad x > a \end{cases}$$

(a) Derive the optimal reconstruction value b as a function of the threshold a for MSE distortion. Express the resulting distortion as function of the threshold a and the variance σ^2.

(b) Determine the decision threshold a in a way that a Lloyd quantizer for MSE distortion is obtained. Determine the distortion and rate for the Lloyd quantizer by assuming fixed-length coding ($R = \log_2 K$) and compare the obtained R-D point with the high-rate approximation.

(c) Can the derived optimal quantizer for fixed-length coding be improved by adding entropy coding (without changing the decision thresholds and reconstruction levels)?

Exercise 3: Lloyd Quantizer for MAE Distortion (Another Alternative)

Given is a stationary source with a zero-mean Laplace pdf $f(x)$ and a symmetric 3-interval quantizer:

$$f(x) = \frac{1}{2m}\, e^{-\frac{|x|}{m}} \qquad \text{and} \qquad Q(x) = \begin{cases} -b & : \quad x < -a \\ 0 & : \quad |x| \leq a \\ b & : \quad x > a \end{cases}$$

(a) Derive the centroid condition and nearest neighbor condition for MAE distortion

$$D = \mathrm{E}\{\, |S - S'| \,\}$$

(b) Derive the optimal reconstruction value b as a function of the threshold a for MAE distortion. Express the resulting distortion as function of the threshold a and the parameter m.

(c) Determine the decision threshold a in a way that a Lloyd quantizer for MAE distortion is obtained. Determine the distortion and rate for the quantizer by assuming fixed-length coding ($R = \log_2 K$).