Exercise 1: Space-Filling Gain for 2-dimensional Vector Quantizer

Calculate the gain (in signal-to-noise ratio) of optimal 2-dimensional vector quantization relative to optimal scalar quantization for high rates on the example of a uniform pdf.

Hints:

- In two dimensions, the optimal quantization cells are regular hexagons; the associated reconstruction vectors are located in the centers of the hexagons.
- For high rates, border effects can be neglected. It can be assumed that the signal space for which the pdf is non-zero is completely filled with regular quantization cells.