Exercise 1: Orthogonal Transforms of Size $N = 2$ (part I)

If we neglect possible reflections of coordinate axes, all orthogonal transforms for 2-d vectors can be specified by

$$\boldsymbol{A} = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

where α is an arbitrary rotation angle.

Consider a zero-mean Gaussian process with variance σ_S^2 and the first-order correlation coefficient ϱ.

(a) Calculate the variances σ_0^2 and σ_1^2 of the resulting transform coefficients as function of ϱ and α.

(b) Calculate the covariance σ_{01}^2 between the resulting transform coefficients as function of ϱ and α.

(c) Consider an even rate distribution $R_0 = R_1 = R$ and determine the associated high-rate distortion-rate function. Does transform coding improve the coding efficiency relative to scalar quantization for this case?

Exercise 1: Orthogonal Transforms of Size $N = 2$ (part II)

(d) Given is the overall rate $R = (R_0 + R_1)/2$. Determine the rate distribution (R_0, R_1) for which the overall distortion $D = (D_0 + D_1)/2$ is minimized (assume that the high rate approximation for scalar quantization of the transform coefficients is valid).

(e) Determine the overall distortion-rate function for optimal rate allocation (and high rates).

(f) Determine the high-rate transform coding gain, which is given by

$$G_T = \frac{D_{\text{scalar quantization}}(R)}{D_{\text{transform coding}}(R)}$$

(g) For what rotation angles is the high-rate transform coding gain maximized (or the distortion minimized)?

Does the optimal rotation angle depend on the correlation coefficient ϱ?

Exercise 2: Implement a PSNR Tool for PPM Images

Implement a tool for measuring PSNRs between two PPM images

- Input to the tool shall be two images in PPM format (original and reconstructed)
- The tool should output the following four Peak-Signal-to-Noise Ratios (PSNR measures)
  - ➡ PSNR of red component, PSNR of green component, PSNR of blue component
  - ➡ Average of the red, green, and blue PSNR

Test the tool by

- Coding one of our test images with JPEG (e.g., using "convert test.ppm test.jpg")
- Reconstructing the JPEG-coded image into the ppm format (e.g., using "convert test.jpg rec.ppm")
- Measuring the PSNRs between the original and reconstructed image using the implemented tool

The PSNR for a color component $c[x, y]$ and its reconstruction $c'[x, y]$ is defined as follows

$$\text{PSNR} = 10 \cdot \log_{10}\left(\frac{255^2}{\text{MSE}}\right) \qquad \text{with} \qquad \text{MSE} = \frac{1}{\text{width} \cdot \text{height}} \sum_{x,y} \left(c'[x, y] - c[x, y]\right)^2$$