Exercise 1: Transform Coding Gain for Gauss-Markov Sources

In the video coding standard ITU-T Rec. H.264 the following forward transform is used:

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & -1 & -2 \\ 1 & -1 & -1 & 1 \\ 1 & -2 & 2 & -1 \end{bmatrix}$$

1. How large is the high-rate transform coding gain (in dB) for a zero-mean Gauss-Markov process with the correlation factor $\rho = 0.9$?
2. By what amount (in dB) can the high-rate transform coding gain be increased if the transform is replaced by a KLT?

NOTE: The basis functions of the given transform are orthogonal to each other, but they don't have the same norm. This has to be taken into account in the calculations.

Exercise 2: High-Rate Bit Allocation for KLT

Consider a zero-mean Gauss-Markov process with variance $\sigma_S^2 = 1$ and correlation coefficient $\rho = 0.9$.
As transform a KLT of size 3 is used, the resulting transform coefficient variances are

$$\sigma_0^2 = 2.7407, \qquad \sigma_1^2 = 0.1900, \qquad \sigma_2^2 = 0.0693$$

Consider high-rate quantization with optimal entropy-constrained scalar quantizers.

1. Derive the high-rate operational distortion rate function.
2. What is the optimal high-rate bit allocation scheme for a given overall rate R?
3. Determine the component rates, the overall distortion, and the SNR for a given overall bit rate R of 4 bit per sample.
4. Determine the high-rate transform coding gain.

Exercise 3: Transform of Image Blocks using the DCT (Implementation)

Prepare a lossy image codec for PPM images. Implement the following:

1. **Reading and writing of PPM images**
   - For details on the PPM format, see older exercises
   - Re-use code from older exercises (see KVV)

2. **Transform coding for sample blocks**
   - a) Apply a separable 8×8 DCT for an image block (or make the block size $N \times N$ variable)
   - b) Quantize the resulting transform coefficient by simple rounding (using a fixed quantization step size)
   - c) Reconstruct transform coefficients (multiplication with quantization step size)
   - d) Apply the inverse transform (inverse DCT)

3. **Test Your Implementation**
   - Apply the transform coding to all sample blocks of an image (without writing a bitstream)
   - ➡ Test the transforms without quantization
   - ➡ Test the transform coding with different quantization step sizes (look at reconstructed images)