Exercise 1: Correlation of Transform Coefficients

Given is a zero-mean AR(1) sources with a variance σ^2 and a correlation coefficient $\varrho = 0.9$

Consider transform coding of blocks of 2 samples using the transform

$$\begin{bmatrix} u_{k,0} \\ u_{k,1} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} s_{2k} \\ s_{2k+1} \end{bmatrix},$$

where k represents the index of the transform block

- Determine the following variances and covariances of the transform coefficients (inside a block and between neighbouring blocks):

$$\begin{aligned} \mathrm{E}\{\, U_{k,0}^2 \,\} &= ? & \mathrm{E}\{\, U_{k,0}\, U_{k+1,0} \,\} &= ? \\ \mathrm{E}\{\, U_{k,1}^2 \,\} &= ? & \mathrm{E}\{\, U_{k,1}\, U_{k+1,1} \,\} &= ? \\ \mathrm{E}\{\, U_{k,0}\, U_{k,1} \,\} &= ? & \mathrm{E}\{\, U_{k,0}\, U_{k+1,1} \,\} &= ? \end{aligned}$$

- Is it worth to exploit the correlations between the transform coefficients of neighboring block (e.g., for typical correlation factors of $\varrho \approx 0.9$) ?

Exercise 2: First Version of Lossy Image Codec (Implementation)

Implement a first lossy image codec for PPM images:

1. Use the source code of last weeks exercise as basis (see KVV)
2. Add some variant of entropy coding for the quantization indexes, for example:
   - Simple Rice coding or Exp-Golomb coding (see lossless codec example in KVV)
   - Adaptive binary arithmetic coding using a unary binarization (see lossless coding example in KVV)
   - ...
3. Implement an encoder that converts a PPM image into a bitstream file
4. Implement a corresponding decoder that converts a bitstream file into a PPM image
5. Test your encoder with some example images and multiple quantization step sizes
6. (Optional) Try to improve your codec by using the YCbCr color format
   - Implement an RBG-to-YCbCr transform before the actual encoding
   - Implement the inverse YCbCr-to-RGB transform after the actual decoding
   - Possible extension: Sub-sampling of chroma components