Predictive Coding

- Prediction
- Prediction in Images
- Principle of Differential Pulse Code Modulation (DPCM)
- DPCM and entropy-constrained scalar quantization
- DPCM and transmission errors
- Adaptive intra-interframe DPCM
- Conditional Replenishment
Prediction

Prediction is difficult – especially for the future.

Mark Twain

- Prediction: Statistical estimation procedure where future random variables are estimated/predicted from past and present observable random variables.

- Prediction from previous samples: 
  \[ \hat{S}_0 = f(S_1, S_2, \ldots, S_N) = f(S) \]

- Optimization criterion
  \[ E = \{ (S_0 - \hat{S}_0)^2 \} = E\{ [S_0 - f(S_1, S_2, \ldots, S_N)]^2 \} \rightarrow \min \]

- Optimum predictor:
  \[ \hat{S}_0 = E\{ S_0 \mid (S_1, S_2, \ldots S_N) \} \]
The optimum predictor \( \hat{S}_0 = E\{S_0 \mid (S_1, S_2, \ldots, S_N) \} \) can be stored in a table (Pixels: 8 bit \( \Rightarrow \) size \( 2^{8N} \)).

Optimal linear prediction (zero mean, Gaussian RVs)

\[
\hat{S}_0 = a_1 S_1 + a_2 S_2 + \ldots + a_N S_N = \mathbf{a}^\mathbf{T} \mathbf{S}
\]

Optimization criterion

\[
E\{(S_0 - \hat{S}_0)^2\} = E\{(S_0 - \mathbf{a}^\mathbf{T} \mathbf{S})^2\}
\]

Optimum linear predictor is solution of

\[
\mathbf{a}^\mathbf{T} \mathbf{R}_S = E\{S_0 \mathbf{S}'\}
\]

In case \( \mathbf{R}_S = E\{\mathbf{S}\mathbf{S}'\} \) is invertible

\[
\mathbf{a} = \mathbf{R}_S^{-1} E\{S_0 \mathbf{S}\}
\]
### Prediction in Images: Intra-frame Prediction

- Past and present observable random variables are prior scanned pixels within that image.
- When scanning from upper left corner to lower right corner:

\[
\begin{array}{ccc}
A & B & C \\
A & X & D
\end{array}
\]

- 1-D Horizontal prediction: A only
- 1-D Vertical prediction: C only
- Improvements for 2-D approaches (requires line store)

\[
\hat{s}(x, y) = \sum_{p=-P_1}^{P_2} \sum_{q=0}^{Q} a(p, q) \cdot s(x - p, y - q)
\]
Prediction Example: Test Pattern

\[
\begin{align*}
    u_H[x,y] &= s[x,y] - 0.95 \cdot s[x-1,y] \\
    u_V[x,y] &= s[x,y] - 0.95 \cdot s[x,y-1] \\
    u_D[x,y] &= s[x,y] - 0.5(s[x,y-1] + s[x-1,y])
\end{align*}
\]
Prediction Example: Cameraman

\[ s[x, y] \]

\[ u_H[x, y] = s[x, y] - 0.95 s[x-1, y] \]

\[ u_V[x, y] = s[x, y] - 0.95 s[x, y-1] \]

\[ u_D[x, y] = s[x, y] - 0.5(s[x, y-1] + s[x-1, y]) \]
Can we use prediction for compression?  
Yes, if we reproduce the prediction signal at the decoder
Differential Pulse Code Modulation

**Coder**

- Input $s$
- Prediction error $e = s - \hat{s}$
- Quantizer $e'$
- Entropy coder $s'$
- Channel $s' - s = e' + e = q$

**Decoder**

- Output $s'$
- Reconstruction $s' = e' + \hat{s}$
- Entropy decoder $e'$
- Predictor $\hat{s}$
DPCM and Quantization

- Prediction is based on quantized samples
- Stability problems for large quantization errors
- Prediction shapes error signal (typical pdfs: Laplacian, generalized Gaussian)
- Simple and efficient: combine with entropy-constrained scalar quantization
- Higher gains: Combine with block entropy coding
- Use a switched predictor
  - Forward adaptation (side information)
  - Backward adaptation (error resilience, accuracy)
- DPCM can also be conducted for vectors
  - Predict vectors (with side information)
  - Quantize prediction error vectors
Comparison for Gauss-Markov Source: $\rho=0.9$

\[
\text{SNR [dB]} = 10 \log_{10} \frac{\sigma^2}{D}
\]

- Linear predictor order $N=1$, $\alpha=0.9$
- Entropy-Constrained Scalar Quantizer with Huffman VLC
- Iterative design algorithm applied
DPCM with Entropy-Constrained Scalar Quantization

Example: Lena, 8 b/p

$K=511, H=4.79 \text{ b/p}$ \hspace{1cm} $K=15, H=1.98 \text{ b/p}$ \hspace{1cm} $K=3, H=0.88 \text{ b/p}$

$K... \text{number of reconstruction levels, } H... \text{entropy}$

from: Ohm
Transmission Errors in a DPCM System

- For a linear DPCM decoder, the transmission error response is superimposed to the reconstructed signal $S'$

- For a stable DPCM decoder, the transmission error response decays

- Finite word-length effects in the decoder can lead to residual errors that do not decay (e.g., limit cycles)
Transmission Errors in a DPCM System II

Example: Lena, 3 b/p (fixed code word length)

Error rate $p=10^{-3}$.

1D pred., hor. $a_H=0.95$  
1D pred., ver. $a_V=0.95$  
2D pred.*, $a_H=a_V=0.5$

from: Ohm
Inter-frame Coding of Video Signals

- Inter-frame coding exploits:
  - Similarity of temporally successive pictures
  - Temporal properties of human vision

- Important inter-frame coding methods:
  - Adaptive intra/inter-frame coding
  - Conditional replenishment
  - Motion-compensating prediction (in Hybrid Video Coding)
  - Motion-compensating interpolation

from: Girod
**Principle of Adaptive Intra/Inter-Frame DPCM**

Predictor is switched between two states:
for moving or changed areas.

**Intra-frame prediction**
for moving or changed areas.

**Inter-frame prediction** (previous frame prediction) for still areas of the picture.

\[
\hat{S} = a_1 \cdot S_1' + a_2 \cdot S_2' + a_3 \cdot S_3' + a_4 \cdot S_4'
\]

\[
\hat{S}_\text{inter} = S'_0
\]

from: Girod
Intra/Inter-Frame DPCM: Adaptation Strategies, I

Feedforward Adaptation

$s \rightarrow e \rightarrow e' \rightarrow VLC \rightarrow e' \rightarrow s'$

$\hat{s}_{intra}$

$\hat{s}_{inter}$

Intraframe predictor

Interframe predictor

Predictor adaptation

Intra/inter-frame Switching information

Coder

Decoder

from: Girod
Intra/Inter-Frame DPCM: Adaptation Strategies, II

Feedback Adaptation

from: Girod
Principle of a Conditional Replenishment Coder

- Still areas: repeat from frame store
- Moving areas: transmit address and waveform

from: Girod
Change Detection

Example of a pel-oriented change detector

- Current frame
  - ABS
  - Average of 3x3 window
  - Threshold
  - Eliminate isolated points or pairs of points
  - Decision changed/unchanged

Example of a block-oriented change detector

- Current frame
  - ABS
  - Accumulate over NxN blocks
  - Threshold
  - Decision changed/unchanged
The "Dirty Window" Effect

Conditional replenishment scheme with change detection threshold set too high leads to the subjective impression of looking through a dirty window.

Moving Area picked up by change detector

Background

Moving areas missed by change detector

from: Girod
Summary

- Prediction: Estimation of random variable from past or present observable random variables
- Optimal prediction
- Optimal linear prediction
- Prediction in images: 1-D vs. 2-D prediction
- DPCM: Prediction from previously coded/transmitted samples (known at coder and decoder)
- DPCM and quantization
- DPCM and transmission errors
- Adaptive Intra/Inter-frame DPCM: forward adaptation vs. backward adaptation
- Conditional Replenishment: Only changed areas of image are transmitted