Predictive Coding

- Prediction
- Prediction in Images
- Principle of Differential Pulse Code Modulation (DPCM)
- DPCM and entropy-constrained scalar quantization
- DPCM and transmission errors
- Adaptive intra-interframe DPCM
- Conditional Replenishment

Prediction

Prediction is difficult – especially for the future.
Mark Twain

- Prediction: *Statistical estimation procedure where future random variables are estimated/predicted from past and present observable random variables.*

- Prediction from previous samples: $\hat{S}_0 = f(S_1, S_2, \ldots, S_N) = f(S)$
- Optimization criterion

$$E = \{(S_0 - \hat{S}_0)^2\} = E\{[S_0 - f(S_1, S_2, \ldots, S_N)]^2\} \rightarrow \min$$

- Optimum predictor:

$$\hat{S}_0 = E\{S_0 \mid (S_1, S_2, \ldots S_N)\}$$

Structure

- The optimum predictor $\hat{S}_0 = E\{S_0 \mid (S_1, S_2, ... S_N)\}$ can be stored in a table (Pixels: 8 bit ➔ size 2^{8N})
- Optimal linear prediction (zero mean, Gaussian RVs)

$$\hat{S}_0 = a_1 S_1 + a_2 S_2 + ... + a_N S_N = \mathbf{a}^t \mathbf{S}$$

- Optimization criterion

$$E\{(S_0 - \hat{S}_0)^2\} = E\{(S_0 - \mathbf{a}^t \mathbf{S})^2\}$$

- Optimum linear predictor is solution of

$$\mathbf{a}^t \mathbf{R}_S = E\{S_0 \mathbf{S}^t\}$$

- In case $\mathbf{R}_S = E\{\mathbf{S}\mathbf{S}^t\}$ is invertible

$$\mathbf{a} = \mathbf{R}_S^{-1} E\{S_0 \mathbf{S}\}$$

Prediction in Images: Intra-frame Prediction

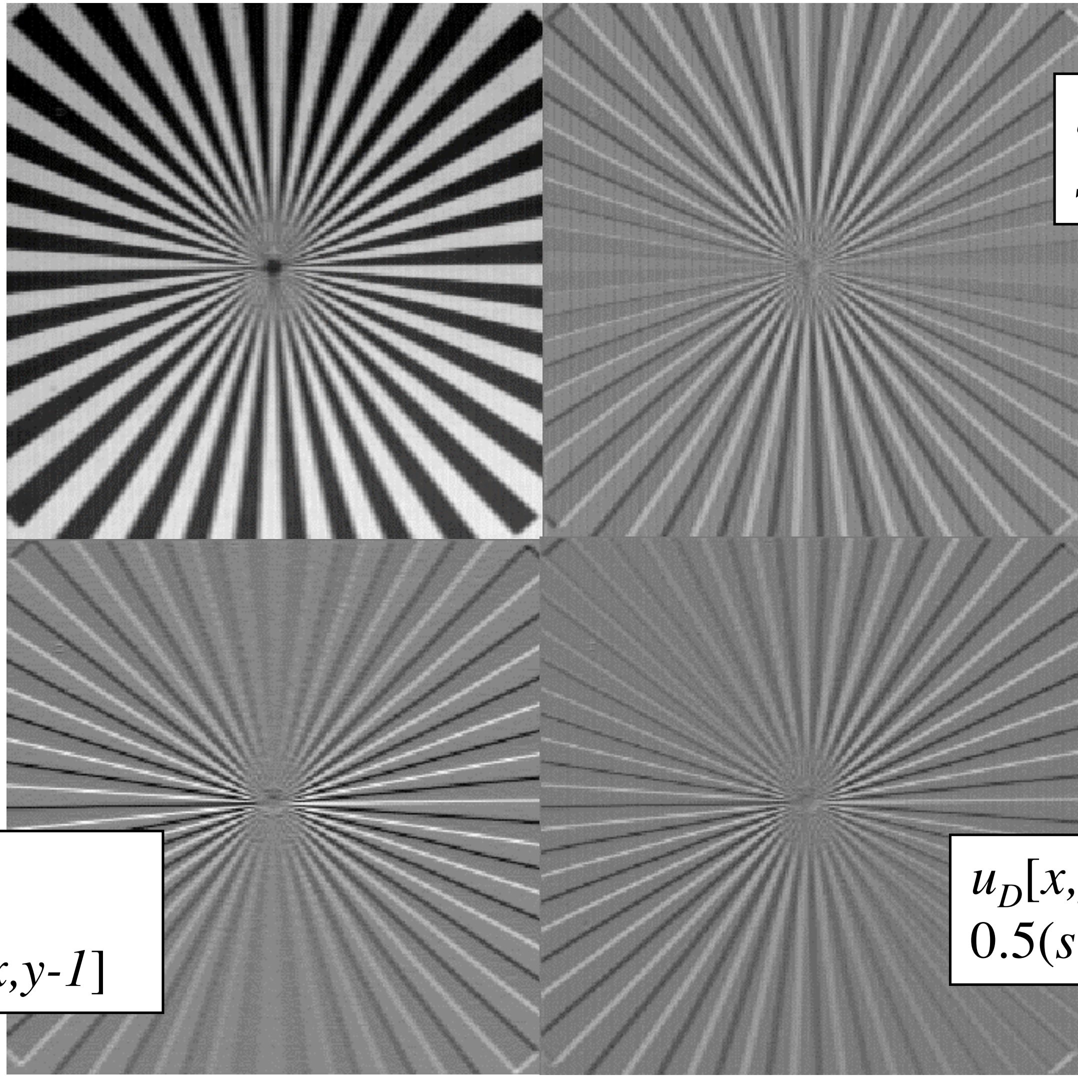
- Past and present observable random variables are prior scanned pixels within that image
- When scanning from upper left corner to lower right corner:

B	C	D
A	X	

- 1-D Horizontal prediction: A only
- 1-D Vertical prediction: C only
- Improvements for 2-D approaches (requires line store)

$$\hat{s}(x,y) = \underbrace{\sum_{p=-P_1}^{P_2} \sum_{q=0}^{Q}}_{(p,q)\neq(0,0)} a(p,q)\cdot s(x-p, y-q)$$

Prediction Example: Test Pattern



$s[x,y]$

$u_H[x,y]= s[x,y]-0.95\ s[x-1,y]$

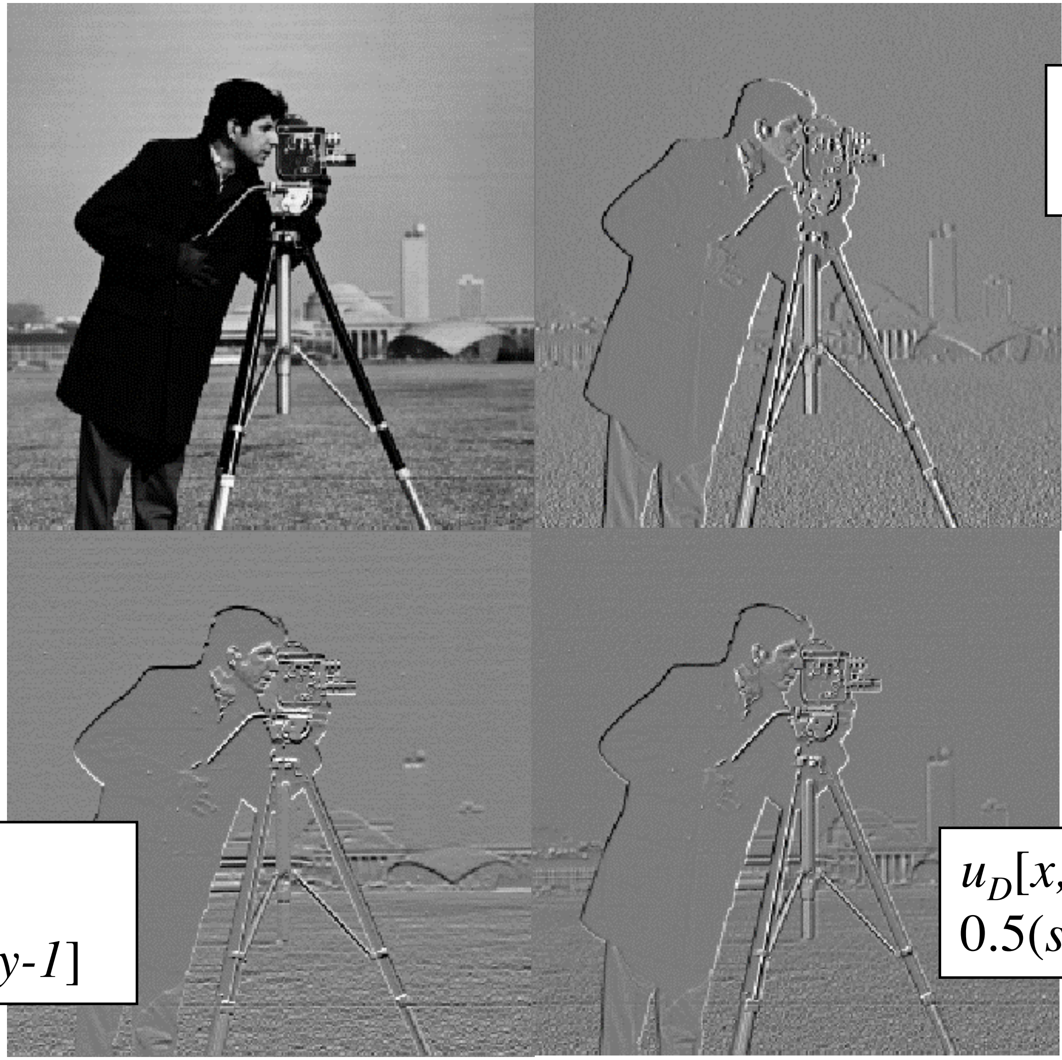
$u_V[x,y]= s[x,y]-0.95\ s[x,y-1]$

$u_D[x,y]=s[x,y]-0.5(s[x,y-1]+ s[x-1,y])$

Prediction Example: Cameraman

$s[x,y]$

$u_H[x,y] = s[x,y] - 0.95\, s[x-1,y]$

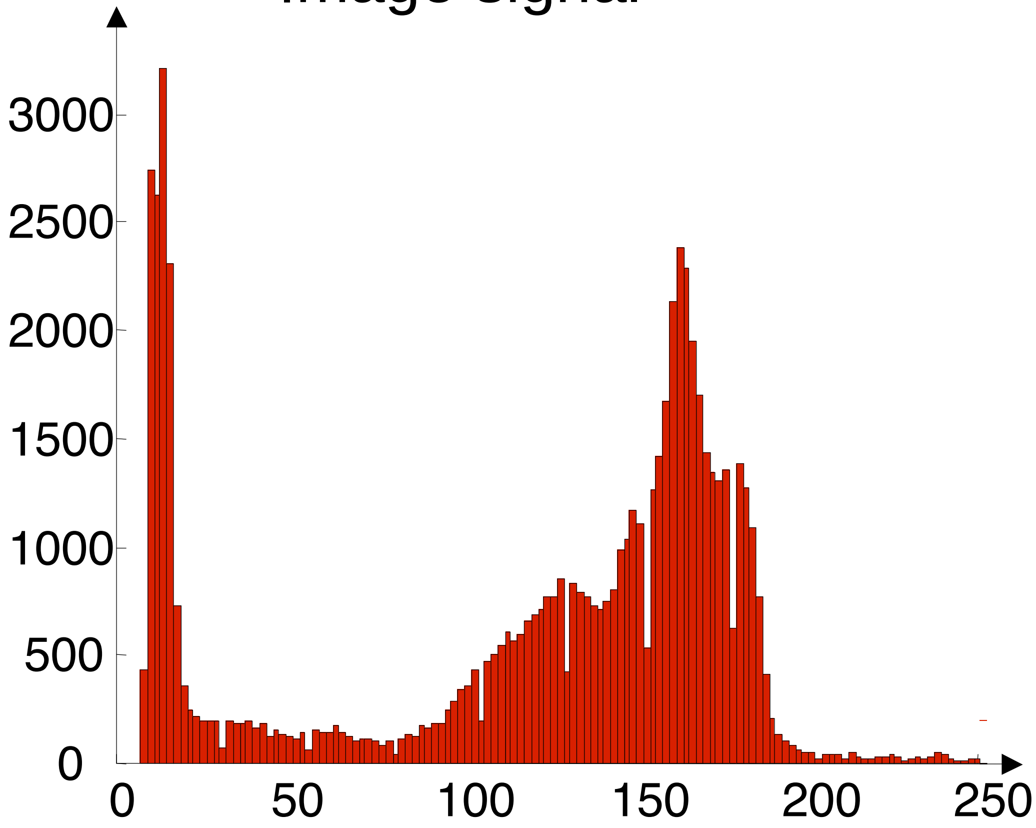


$u_V[x,y] = s[x,y] - 0.95\, s[x,y-1]$

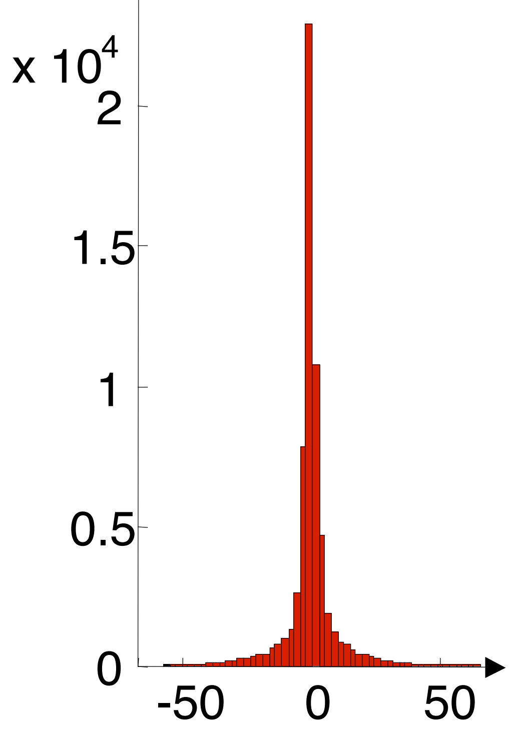
$u_D[x,y] = s[x,y] - 0.5(s[x,y-1] + s[x-1,y])$

Change of Histograms: Cameraman

Image signal



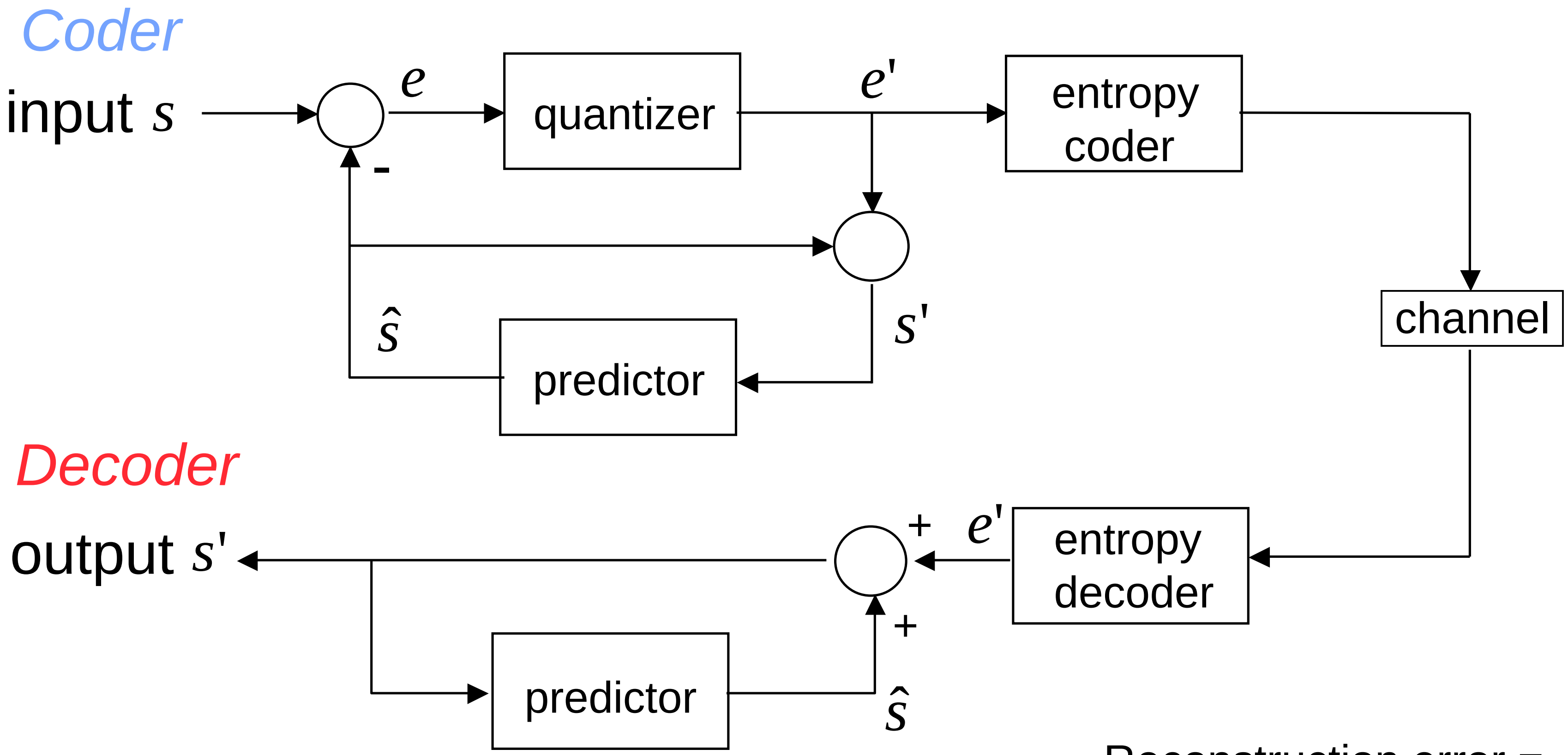
Prediction error signal (diag. pred.)



Can we use prediction for compression ?

Yes, if we reproduce the prediction signal at the decoder

Differential Pulse Code Modulation



Prediction error

$$e = s - \hat{s}$$

Reconstruction

$$s' = e' + \hat{s}$$

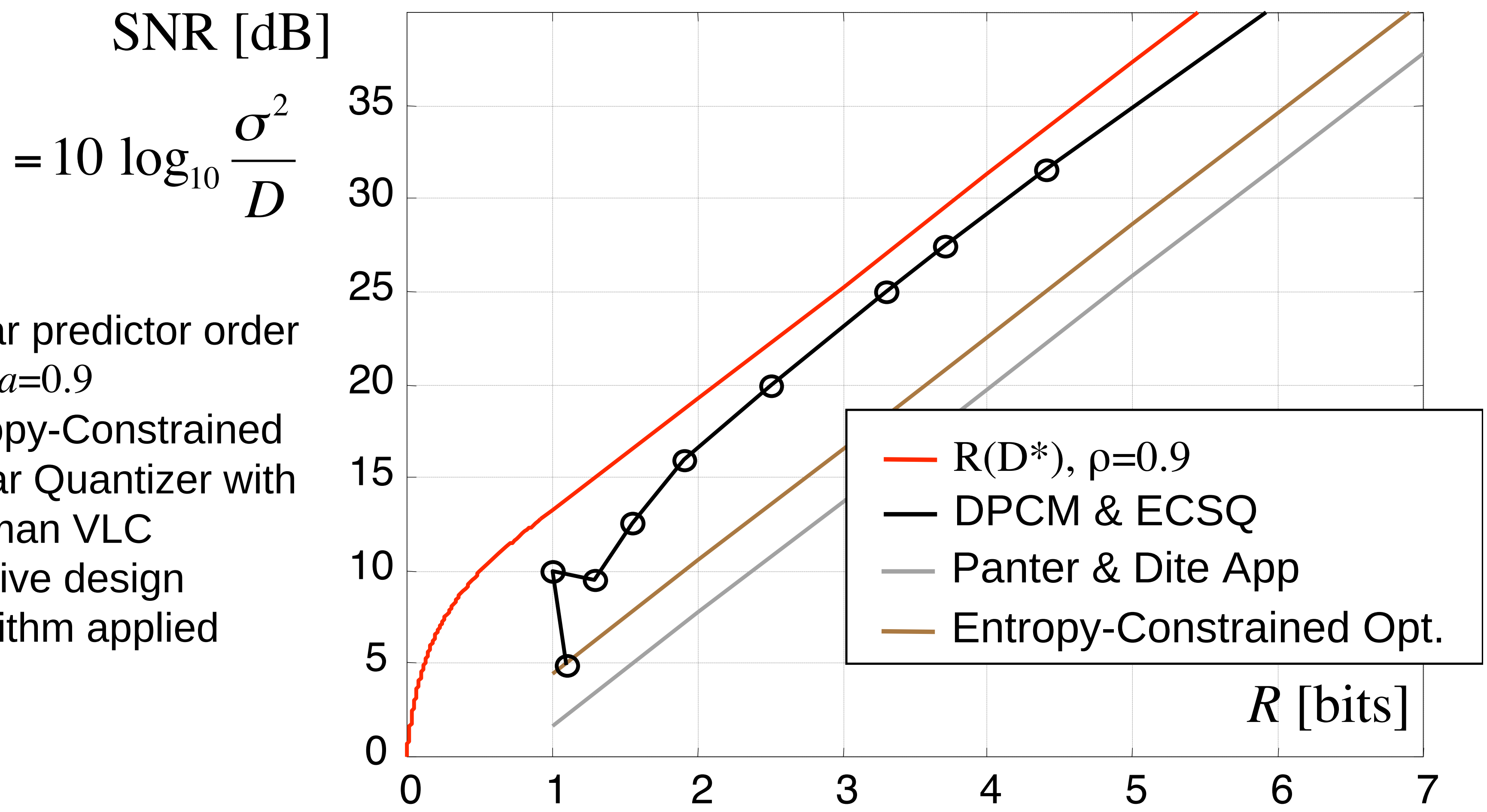
Reconstruction error = quantization error

$$s' - s = e' - e = q$$

DPCM and Quantization

- Prediction is based on quantized samples
- Stability problems for large quantization errors
- Prediction shapes error signal (typical pdfs: Laplacian, generalized Gaussian)
- Simple and efficient: combine with entropy-constrained scalar quantization
- Higher gains: Combine with block entropy coding
- Use a switched predictor
  - Forward adaptation (side information)
  - Backward adaptation (error resilience, accuracy)
- DPCM can also be conducted for vectors
  - Predict vectors (with side information)
  - Quantize prediction error vectors

Comparison for Gauss-Markov Source: ρ=0.9



- Linear predictor order $N=1$, $a=0.9$
- Entropy-Constrained Scalar Quantizer with Huffman VLC
- Iterative design algorithm applied

DPCM with Entropy-Constrained Scalar Quantization

Example: Lena, 8 *b*/*p*

K=511, H=4.79 *b*/*p* K=15, H=1.98 *b*/*p* K=3, H=0.88 *b*/*p*

K...number of reconstruction levels, H...entropy

from: Ohm

Transmission Errors in a DPCM System

- For a linear DPCM decoder, the transmission error response is superimposed to the reconstructed signal S'
- For a stable DPCM decoder, the transmission error response decays
- Finite word-length effects in the decoder can lead to residual errors that do not decay (e.g., limit cycles)

from: Girod

Transmission Errors in a DPCM System II

Example: Lena, 3 b/p (fixed code word length)

Error rate $p=10^{-3}$.

1D pred., hor. $a_H=0.95$

1D pred., ver. $a_V=0.95$

2D pred.*, $a_H=a_V=0.5$

from: Ohm

Inter-frame Coding of Video Signals

- Inter-frame coding exploits:
  - Similarity of temporally successive pictures
  - Temporal properties of human vision

- Important inter-frame coding methods:
  - Adaptive intra/inter-frame coding
  - Conditional replenishment
  - Motion-compensating prediction (in Hybrid Video Coding)
  - Motion-compensating interpolation

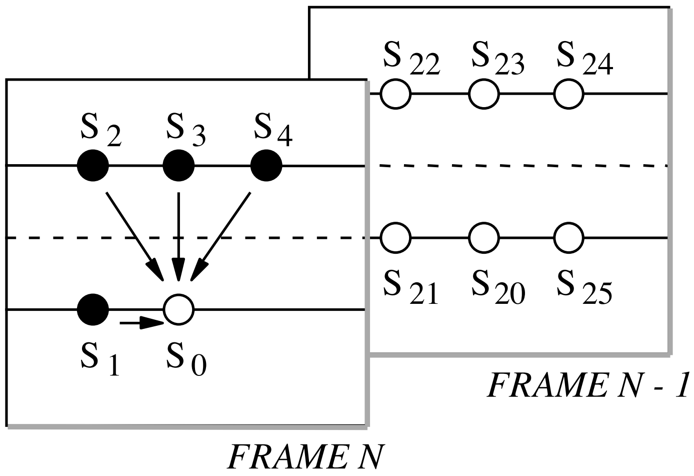
from: Girod

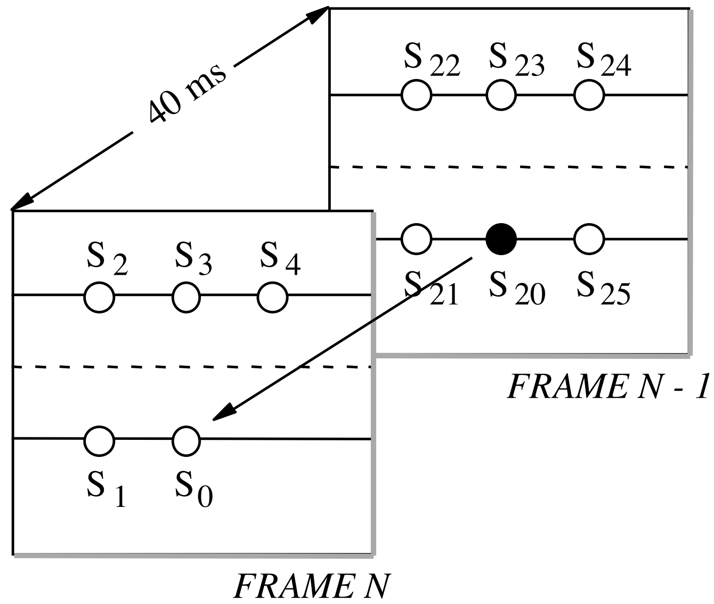
Principle of Adaptive Intra/Inter-Frame DPCM

Predictor is switched between two states:
for moving or changed areas.

Intra-frame prediction
for moving or changed areas.

Inter-frame prediction (previous frame prediction) for still areas of the picture.





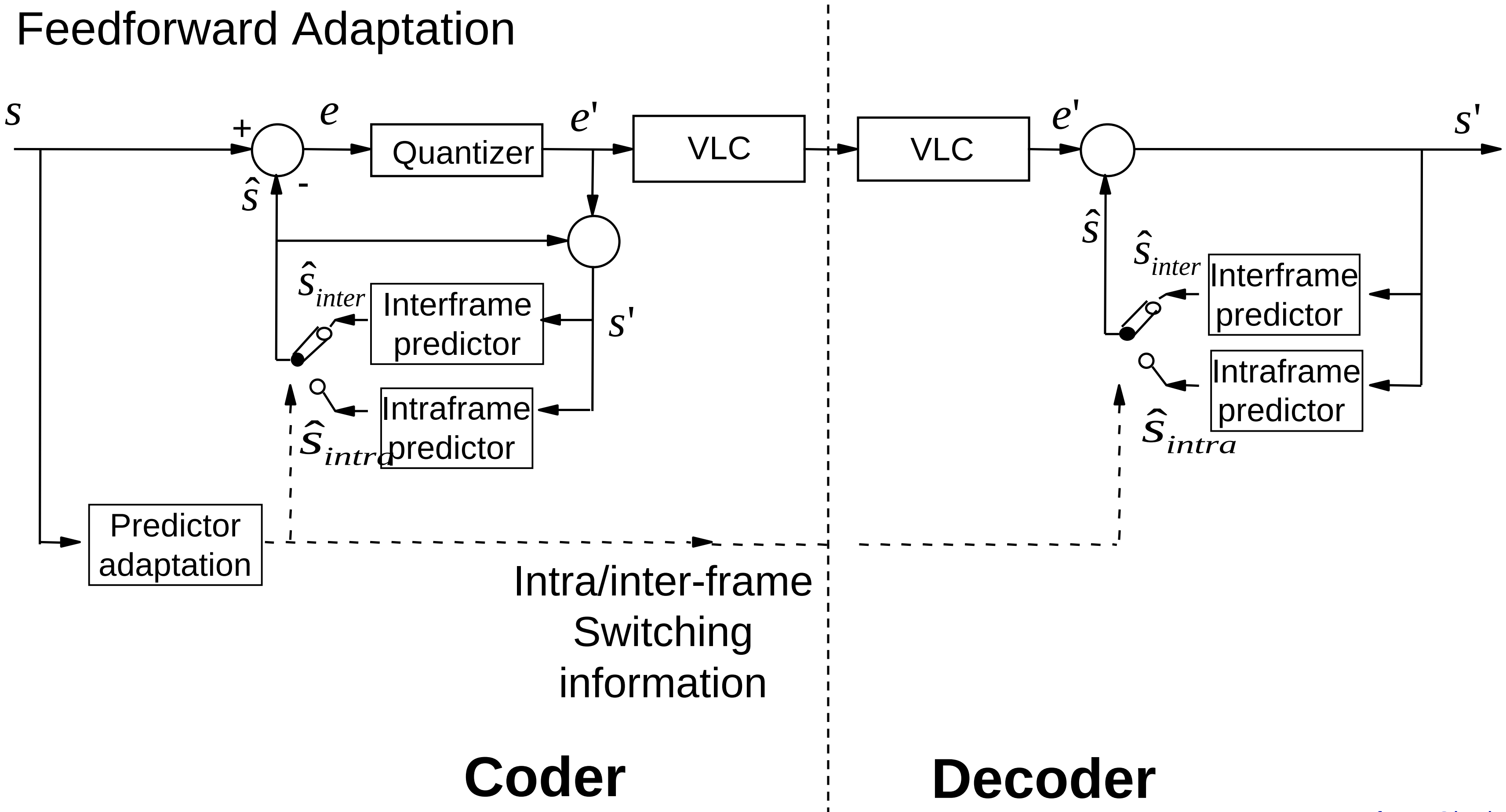
$$\hat{S} = a_1 \cdot S_1' + a_2 \cdot S_2' + a_3 \cdot S_3' + a_4 \cdot S_4'$$

$$\hat{S}_{inter} = S'_{20}$$

from: Girod

Intra/Inter-Frame DPCM: Adaptation Strategies, I

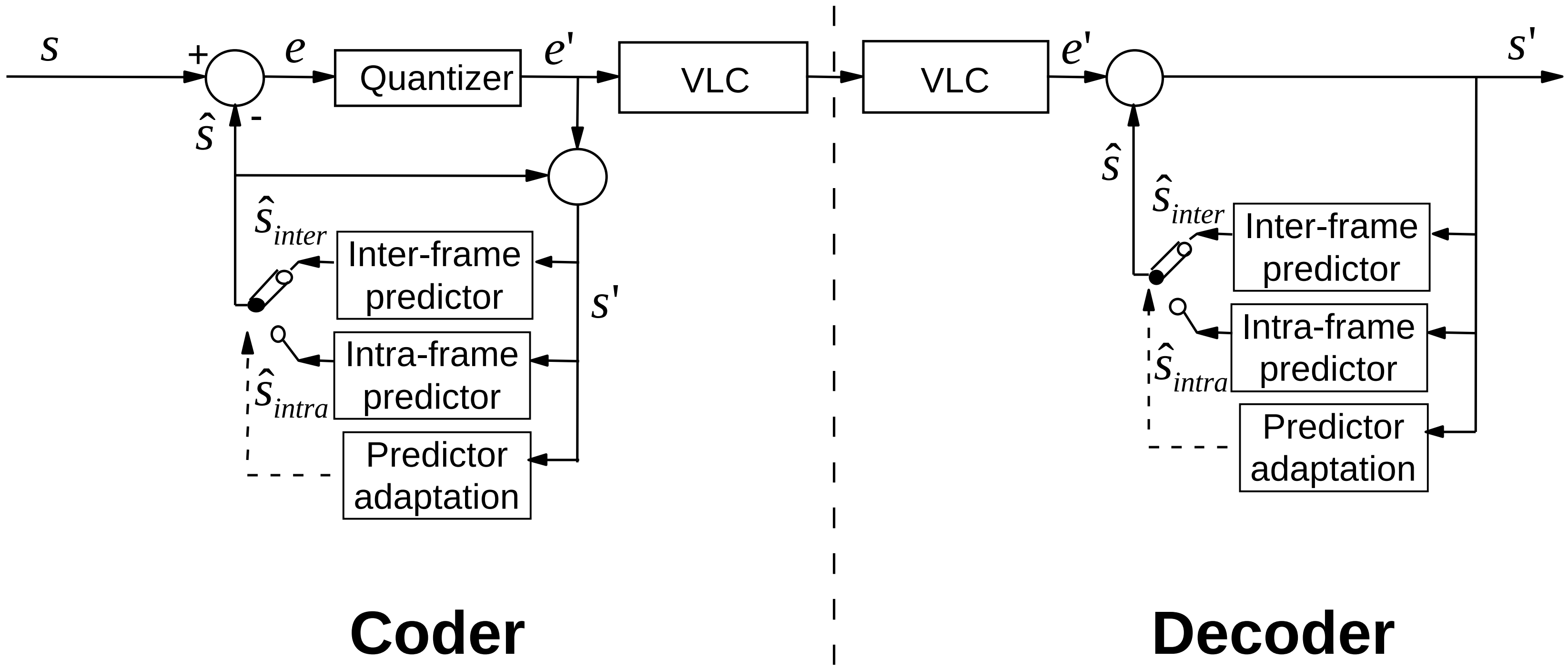
Feedforward Adaptation



from: Girod

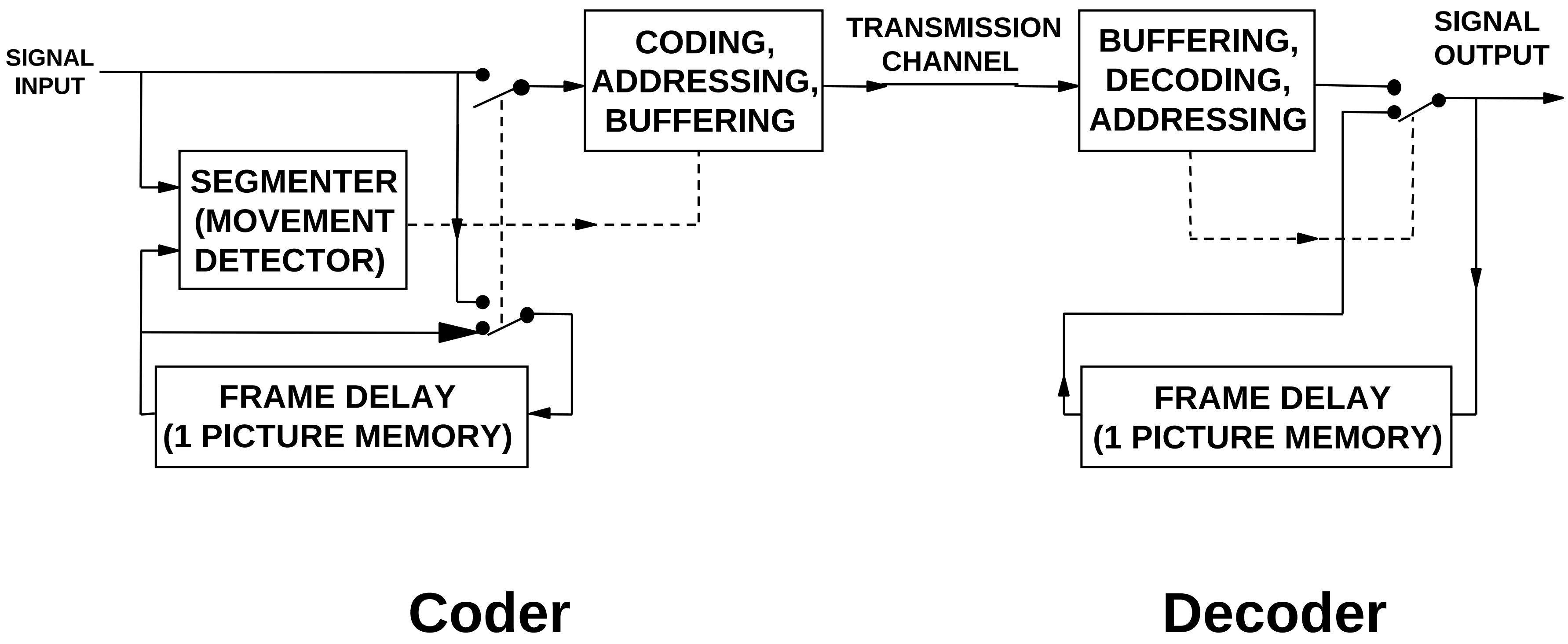
Intra/Inter-Frame DPCM: Adaptation Strategies, II

Feedback Adaptation



from: Girod

Principle of a Conditional Replenishment Coder

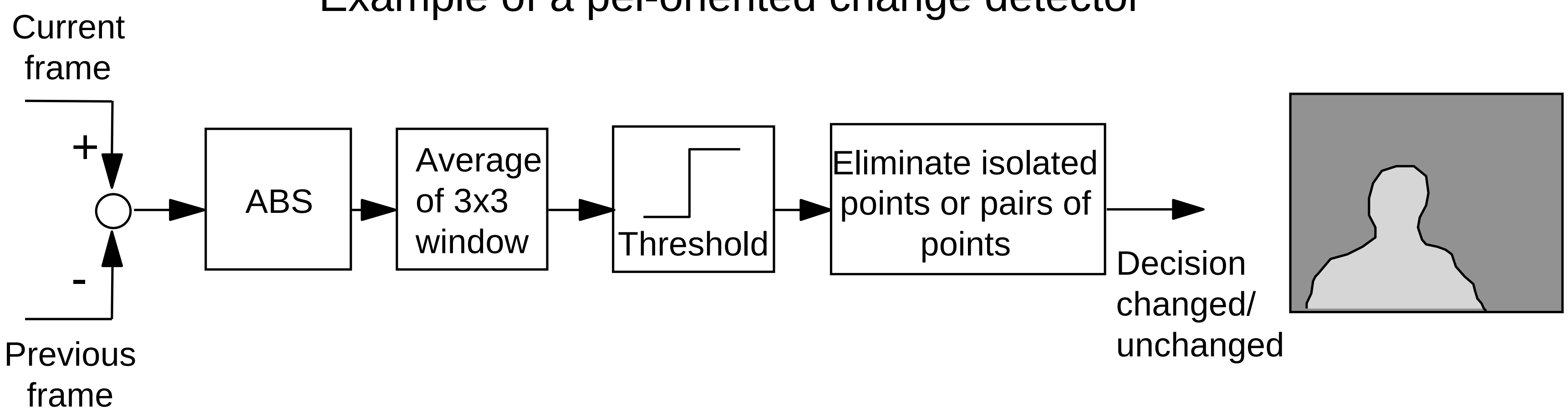


- Still areas: repeat from frame store
- Moving areas: transmit address and waveform

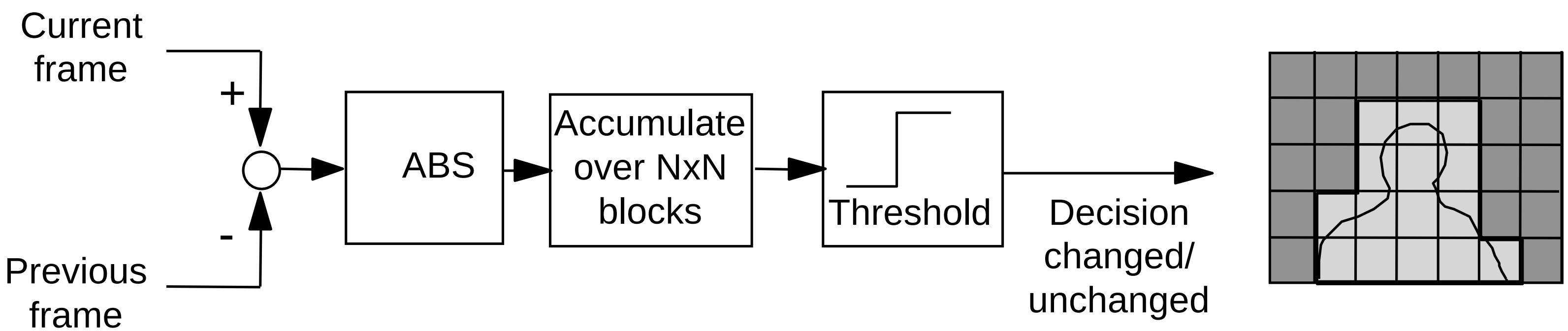
from: Girod

Change Detection

Example of a pel-oriented change detector



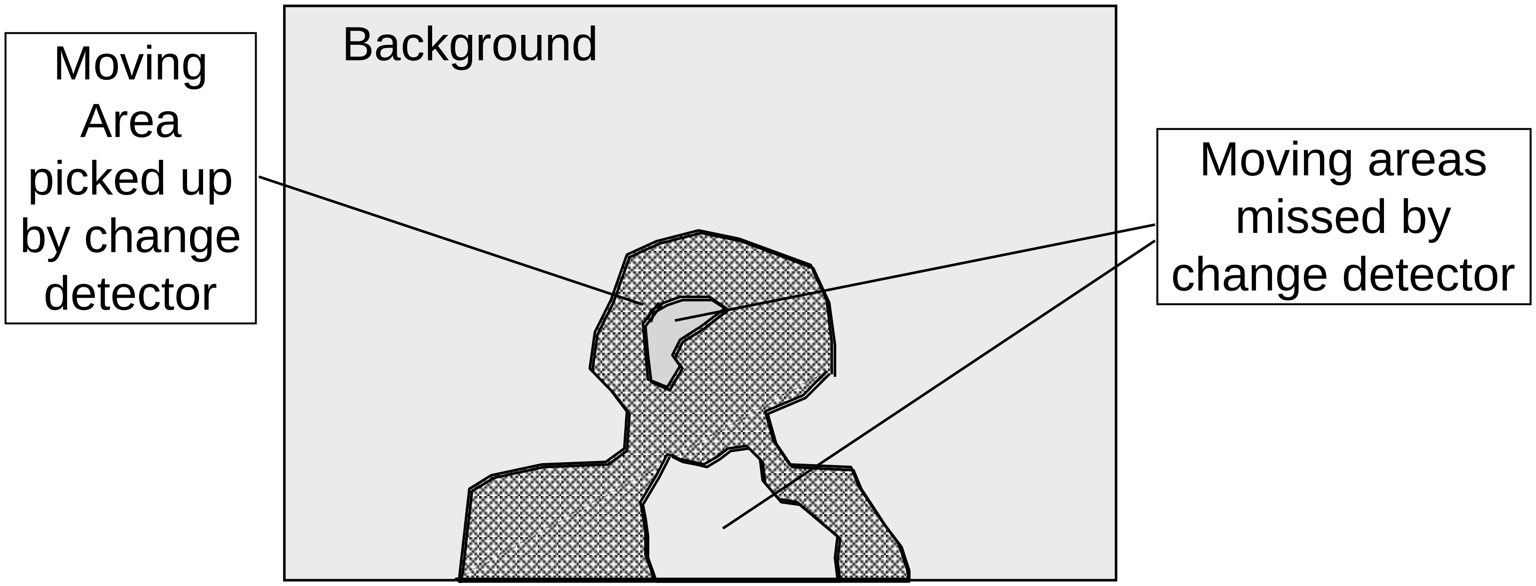
Example of a block-oriented change detector



from: Girod

The "Dirty Window" Effect

Conditional replenishment scheme with change detection threshold set too high leads to the subjective impression of looking through a dirty window.



from: Girod

Summary

- Prediction: Estimation of random variable from past or present observable random variables
- Optimal prediction
- Optimal linear prediction
- Prediction in images: 1-D vs. 2-D prediction
- DPCM: Prediction from previously coded/transmitted samples (known at coder and decoder)
- DPCM and quantization
- DPCM and transmission errors
- Adaptive Intra/Inter-frame DPCM: forward adaptation vs. backward adaptation
- Conditional Replenishment: Only changed areas of image are transmitted