

# Group-wise Stationary Subspace Analysis - A novel method for studying non-stationarities

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## Abstract

In this paper we present an extension of the recently proposed Stationary Subspace Analysis (SSA). This novel method solves the problem how to group signals from different conditions and/or subjects to find stationary subspaces. The original SSA approach does not offer a natural way to group data and therefore better define the non-stationarities of interest. This drawback is solved with group-wise SSA (gwSSA) and demonstrated with a simple but illustrative example: the classification of BCI data. If not treated correctly the BCI tasks are considered as non-stationarities in SSA, which complicates its use for classification purposes. We show how, by correctly defining groups, non-stationarities of interest can be extracted. In this paper, the application is in multi-class signals, where the groups are properly defined to even improve classification performance.

## 1 Introduction

In Brain-Computer Interfacing (BCI) [1] one major challenge is to understand the non-stationarities in the signal of interest e.g. EEG and to develop methods which are invariant to them. The sources and time scales of non-stationarities in the signal can be very different e.g. changes in electrode impedance may occur when an electrode gets loose or the gel dries out, muscular activity or eye movements lead to artefacts in the signal and we often observe changes of task involvement as subjects get tired. Further changes in the EEG can be caused by differences between sessions e.g. no feedback in the calibration session vs. feedback in later sessions (or different kinds of feedback) or small differences in electrode positions between sessions. Several approaches were proposed to reduce the impact of non-stationarities in BCI applications. For example [2, 3, 4, 5] use techniques for co-adaptive learning of user and machine, [6] uses extra measurement like EOG or EMG to be invariant against muscular or ocular artefacts and [7] applies covariate shift adaptation to account for changes of the features. Recently, Bünau et al. [8] proposed a novel technique called Stationary Subspace Analysis (SSA) which finds the low-dimensional projections having stationary distributions from high-dimensional observations. This method can be applied to EEG data as a preprocessing step in order to extract the stationary part of the signal as done in [9]. The authors showed that restricting the BCI to the stationary sources found by SSA can significantly increase the classification accuracy. However, SSA is a general purpose method and its usage is limited when applying it to multi-class data because the distinctive different tasks can be considered as

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non-stationary components of the signal (it is expected that the statistical properties of the data change with the task) and therefore disregarded. To avoid this, the data needs to be carefully preprocessed. In this paper we extend the work of Bünau et al. [9] and propose a group-wise Stationary Subspace Analysis (gwSSA) method which allows to compute the stationary subspace from different conditions and/or subjects. We analyse the emerging stationarity and non-stationarity patterns obtained from five volunteers and show that our method is better suited for multi-class data and consequently outperforms SSA.

This paper is organized as follows. In the next section we present SSA and introduce our group-wise approach. After that in Section 3 we apply it to a dataset of five subjects performing motor imagery and analyse the results in Section 4. We conclude in Section 5 with a discussion.

## 2 Group-wise Stationary Subspace Analysis

Stationary Subspace Analysis (SSA) [8] is a novel method to factorize a high-dimensional multivariate time-series into its stationary and non-stationary components. Its underlying assumption is that the observed signal  $\mathbf{x}(t)$  is a linear superposition of the stationary  $\mathbf{s}^s(t)$  and non-stationary  $\mathbf{s}^n(t)$  sources

$$\mathbf{x}(t) = A \mathbf{s}(t) = [A^s \quad A^n] \begin{bmatrix} \mathbf{s}^s(t) \\ \mathbf{s}^n(t) \end{bmatrix}, \quad (1)$$

and  $A$  is an invertible matrix. The goal of SSA is to find a linear transformation  $\hat{A}^{-1}$  that separates the  $\mathbf{s}$ -sources from the  $\mathbf{n}$ -sources. For that the signal  $\mathbf{x}(t)$  is divided into epochs and an optimization criterion is employed to recover the sources. More precisely, SSA minimizes the distance measured as Kullback-Leibler-Divergence  $D_{\text{KL}}$ , between the distribution of the estimated  $\mathbf{s}$ -sources in each epoch (described by first two moments) and the standard normal distribution.

The idea behind group-wise SSA (gwSSA) is to consider groups of epochs<sup>1</sup> in order to find projections which are as stationary as possible in each group. This does not necessarily imply stationarity across all epochs, however, it has the important advantage that one can combine data from many subjects to conduct group studies or one can group different conditions for a single subject, for example for classification. Since the objective function of gwSSA measures the distances between the distribution of the epoch and the mean distribution of the group, it can be written as

$$L(R) = \sum_{i=1}^M \sum_{j=1}^{N_i} D_{\text{KL}} \left[ \mathcal{N}(\hat{\boldsymbol{\mu}}_{ij}^s, \hat{\Sigma}_{ij}^s) \parallel \mathcal{N}(\bar{\boldsymbol{\mu}}_j^s, \bar{\Sigma}_j^s) \right], \quad (2)$$

where  $M$  is the number of groups,  $N_i$  is the number of epochs in group  $i$ ,  $\mathcal{N}(\hat{\boldsymbol{\mu}}_{ij}^s, \hat{\Sigma}_{ij}^s)$  is the distribution of epoch  $j$  in group  $i$  and  $\mathcal{N}(\bar{\boldsymbol{\mu}}_j^s, \bar{\Sigma}_j^s)$  is the average distribution in group  $i$ . As with SSA it is possible minimize the objective function by conjugate gradient descend in the space of antisymmetric matrices (see [8] for details).

## 3 Data

The data used in this paper consists of two calibration (i.e. without feedback) recordings from five healthy participants. The volunteers performed motor imagery of two limbs, specifically 'left hand' and 'foot'. The cues were presented either visually (with an arrow appearing in the centre of the screen) or auditory (a voice announcing the task to be performed), resulting in two different datasets for each user. In this experiment, the training data was the calibration with visual stimuli and the testing data, the calibration with auditory stimuli. The preprocessing parameters (frequency band and time interval) were subject-optimized in the training test, which consisted of 132 trials (equally distributed for each class). The testing set contained the same number of trials as the training dataset. The data was recorded with a multichannel system of 85 electrodes densely

<sup>1</sup>Epochs can be grouped according to different criteria e.g. each subject and/or condition can represent a group.

| Methods / Subjects            | Subject 1   | Subject 2   | Subject 3   | Subject 4   | Subject 5   |
|-------------------------------|-------------|-------------|-------------|-------------|-------------|
| Without SSA                   | 90.9        | <b>80.0</b> | 73.3        | 70.8        | 94.2        |
| SSA (epoch=1 trial)           | 90.9        | 60.0        | <b>82.5</b> | 70.8        | 82.5        |
| SSA (epoch=1 trial per class) | 87.8        | 75.8        | 77.5        | 74.1        | 93.3        |
| gwSSA (groups=classes)        | <b>91.7</b> | 78.3        | 80.0        | <b>77.5</b> | <b>97.5</b> |

Table 1: Comparison of classification accuracies for five subjects performing motor imagery. The SSA-based methods are applied as preprocessing step i.e. the bandpass-filtered EEG data are projected to the stationary subspace before performing CSP. The target dimensionality is selected via 5-fold cross-validation on the training data. From the results we see that our group-wise SSA method performs better than the other approaches, especially on subject 3, 4 and 5. The direct application of SSA (without trial concatenation) performs very poorly on subject 2 and 5, probably because it removes the differences between the classes.

covering the motor cortex at 1000 Hz. After filtering, it was down-sampled to 100 Hz. The features are extracted using log-band power on CSP filtered channels. The CSP filters were computed in the training test (three filters were selected per class). Finally, the classifier was Linear Discriminant Analysis (LDA). In the case of applying SSA or gwSSA, the band-pass filtered EEG data of the training set was used to feed the algorithm. The data was projected in the resulting stationary dimensions and after that the same feature extraction method as explained above was applied. SSA and gwSSA were restarted 50 times in order to avoid local minima and the dimensionality of the stationary subspace was selected via 5-fold cross-validation on the training set.

## 4 Results

In this section we compare our group-wise SSA to three baselines, namely SSA which takes every trial as an epoch, SSA which combines trials of opposite classes (as done in [9]) and no SSA. We apply gwSSA to each subject using two groups i.e. one group consist of trials which are labeled as 'left hand' whereas all 'foot' trials are in the other group. This is to assure that differences between both classes are not treated as non-stationarities and ignored. In Table 1 we see that gwSSA greatly improves the classification accuracies of subject 3, 4 and 5 while leaving the other subjects more or less at baseline level. Although SSA with trial combination also leads to higher classification accuracies in those subjects, it performs worse than gwSSA. We conjecture that this is because our method solves the problem in a principled way and does not require heuristics such as concatenation of trials with opposing labels. As expected the direct application of SSA (without trial combination) performs poorly as important differences between both classes may be treated as non-stationarities. In order to understand why group-wise SSA improves classification performance, we consider the changes which occur between training and test phase. In Figure 1 we plot the mean differences in power between the training and test features. Since we provided the visual cue only in the training phase, we observe large changes in the occipital areas. Although our SSA approach is computed on training data only, it greatly reduces these changes providing more stationary features. The most significant noise reduction can be observed for subjects who improve the most. By analysing the changes in power for each trial we can identify the locations of non-stationarities. These locations vary between subjects and can partially explain the results difference between subjects 1, 2 and 3, 4, 5. The first two subjects do not benefit from applying SSA as non-stationarities are located in areas which are important for discrimination (i.e. important information may be discarded), whereas in the other group the changes lie in the occipital area.

## 5 Conclusion

We presented an extension of the SSA algorithm which can identify stationary brain sources from groups of subjects and/or conditions. We showed that our group-wise SSA method can be naturally

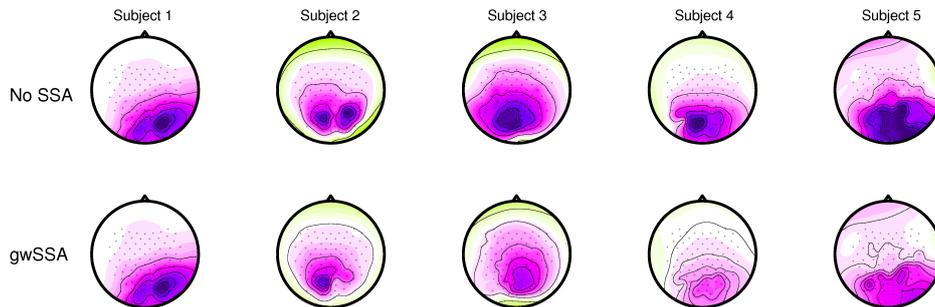


Figure 1: Scalp plots showing the mean difference in power between the training and the test data. When no preprocessing is applied there is a significant change between training and test features, most likely because the cue is showed with different types of stimuli (visual and auditory). Our approach reduces the shift, thus makes the signal more stationary but preserves the class related information contained in the signal. The effect is especially large for the subjects 3 and 4.

applied to multi-class signals and does not rely on heuristics to optimize its use. One important drawback of the heuristic approach applied in [9] is that it cannot be applied when the classes are not equally distributed. However, group-wise SSA solves the grouping problem in a principled way. Furthermore, it outperforms the standard SSA approach and the no SSA baseline for 3 out of 5 users. Finally, gwSSA can also be applied to group studies, because it optimizes stationarity in groups irrespectively of the grouping criterion. This means that stationary properties can be found in neuro-scientific data by simply defining the group of subjects as the target. In the future we want to use this tool to show how to analyze EEG data in group studies.

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